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Analysis of continuous space frames by three-dimensional moment-distribution and slope-deflection methods

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ANALYSIS OF CONTINUOUS SPACE FRAMES BY THREE-DIMENSIONAL MOMENT-DISTRIBUTION AND SLOPE-DEFLECTION METHODS

by

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I. INTRODUCTION

A. Historical Review

Structural engineering, like any other science has been undergoing changes gradually. In the early days structures were built more or less by empirical methods based upon previous experience. The structures so built were, as a rule, much stronger than they should be. This practice resulted in uneconomical use of materials. Moreover, designs based on broad assumptions or limited experience are not always reliable. Therefore, some principles should be developed upon which to base judgment instead of depending entirely upon personal feelings which are affected by the senses of the individual.

The first attempt engineers made toward the establishment of principles was to subdivide a structure into planar units and to assume forces acting on these planar structural units to be lying in the same plane. The effect of members lying outside of the plane is neglected entirely. However, as everyone knows very few structures lie in one plane nor do the forces acting on them lie exclusively in planes.

It is only natural that some engineers, as eager students who seek the truth, should begin to investigate structures as they are, that is as structures in space. When the entire three-dimensional structural frame is considered as the unit for analysis it is called a Space Frame.

Several methods have been developed for solving the particular type of space frames whose members are subjected primarily to compressive and tensile stresses. Structures which belong to this group may be called Trussed Space Frames.

Since the introduction of reinforced concrete structures and the adoption of welding for steel work, engineers are confronted with the problem of continuity among the elements of a structure. Here again engineers have followed the natural course of cutting the structure as well as the forces acting upon it into co-planar units.

The author, inspired by the space or three-dimensional idea has undertaken the development of some methods to solve this type of structure. Structures consisting of beams and columns which are continuous in three dimensions may be called Continuous Space Frames. The members of a continuous space frame are subjected primarily to flexural stresses although direct axial stresses occur to a lesser degree.

Two standard methods, moment-distribution (1) and slope-deflection (2) have been extended by the author to solve continuous space frames. These analyses are called Three-Dimensional Moment-Distribution and Slope-Deflection Methods. It is interesting to know that as a by-product of this study some of the problems (3) which heretofore were not solvable by the direct application of the moment-distribution method can be solved now. It can be shown that there is no need for the distinction between the so-called "in series" and "in parallel" arrangements (4).

As may be expected fewer assumptions are made in the solution of space frames than for structures cut into planar units. Consequently, the solution gives comparatively exact picture of what is happening in the actual structure. Of course, some engineers might be very reluctant in applying these methods to practical problems. However, when some unusual or unexpected problems occur, either of these methods provides a means for obtaining information that should lead to rational explanations for the behavior of the structure. The methods as herein developed should also serve to study the basic behavior of continuous space frames subjected to lateral as well as gravitational forces.

The need for a method to solve continuous space frame problems is demonstrated by the general approximations (5) the committee had used to explain the behavior of the structural frame. The author wishes to call the reader's attention to the fact that those problems can be handled to advantage by the use of the methods described in the following chapters.

It can be said here that these methods are applicable to all continuous space frames as long as the members meet at right angles to each other.

In the discussion of the three-dimensional methods it is assumed that the readers are acquainted with the moment-distribution and slope-deflection methods applied to planar structures.

B. Classification of Structural Types

For the purpose of this thesis, frames used in structures can be divided into two main classes, trussed frames and continuous frames. Each of them can be subdivided into planar frames and space frames. Trussed frames can be statically determinate or statically indeterminate. Continuous frames are, however, all statically indeterminate.

The following classification will show all these types:

- Structural types
  - Trussed
    - Planar
      - Statically determinate
      - Statically indeterminate
    - Space
      - Statically determinate
      - Statically indeterminate
  - Continuous
    - Planar-statically indeterminate
    - Space-statically indeterminate

C. Classification of Continuous Frames

For the purposes of analysis, continuous frames can be classified according to the number of possible directions of displacement:

1. No direction of displacement
2. One direction of displacement
3. Two directions of displacement
4. Three directions of displacement

Typical frames belonging to this classification are shown in Figure 1. With the exception of Class 4, all the other classes may have both planar

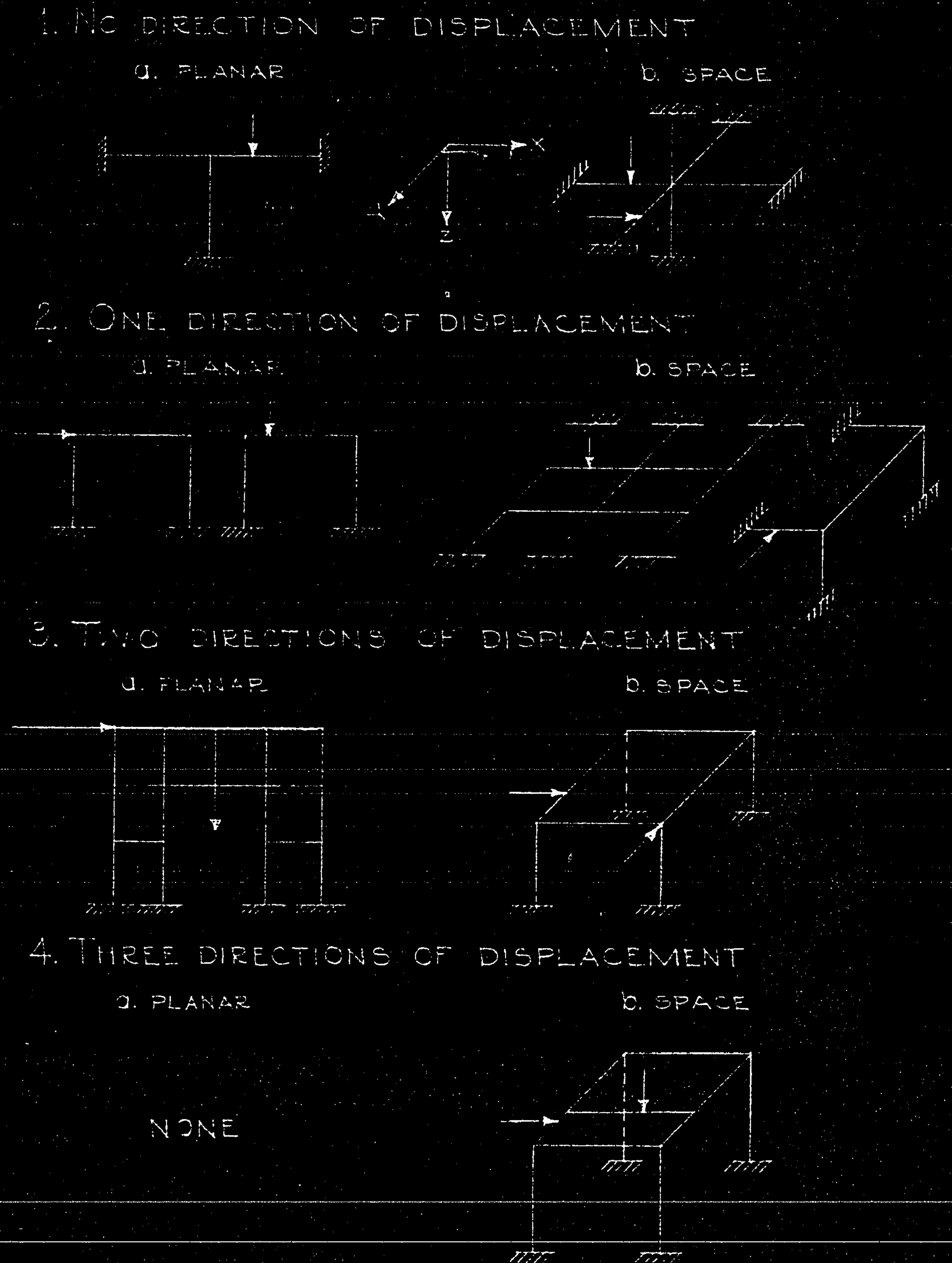


FIG. 1 CLASSIFICATION OF CONTINUOUS FRAMES

and space applications. The conditions in Class 4 can only be satisfied by space frames.

D. Assumptions

As stated before fewer assumptions are made in space frame analyses than in planar frames. The following list gives the important assumptions upon which the development of the method of solution is based:

1. The angles between members at the joint do not change. When the joint rotates the end of each member framing into the joint rotates by the same amount.

2. The change of length due to direct stress is neglected. When two parallel members are joined at right angles by a third member, the deflections of the parallel members along the axis of the third are equal.

3. The effect of the "warping" (6) of the cross section due to torsion is neglected.

4. The effect of the change of moment of inertia due to the shifting of the neutral axis caused by torsion is neglected.

5. The principle of superposition holds.

6. The conditions of equilibrium which apply to the structure as a whole, to each joint, and to each member are:

$$\Sigma F_x = 0, \quad \Sigma F_y = 0, \quad \Sigma F_z = 0$$

$$\Sigma M_x = 0, \quad \Sigma M_y = 0, \quad \Sigma M_z = 0$$

7. The materials are assumed, as usual, to be isotropic, homogeneous and elastic.

II. THREE-DIMENSIONAL MOMENT-DISTRIBUTION METHOD

A. Nomenclature and Conventional Signs

In space frames all moments and shears are given with reference to the axes X, Y, and Z. To facilitate the work the following nomenclature and conventional signs are used.

1. M_X, M_Y, and M_Z are moments with respect to axes X, Y, and Z respectively. They act upon the structure in the planes perpendicular to the axes X, Y, and Z respectively. They are considered to be positive when they tend to rotate the joint in a clockwise direction as one looks along the positive directions of the axes X, Y, and Z respectively.
2. M_X^*, M_Y^*, and M_Z^* are torques with respect to the axes X, Y, and Z respectively. In the tables, the torques are indicated by placing an asterisk at the upper right corner of the member as AB^*. These torques are considered to be positive when they tend to rotate the joint in a clockwise direction as one looks along the positive direction of the axes X, Y, and Z respectively. Members parallel to the axes X, Y, and Z are under torsion with respect to the axes X, Y, and Z respectively.
3. SM_{AX}, SM_{AY}, SM_{AZ}, are shear moments for joints A, B, C, ... at which the shears act along axes X, Y, and Z respectively.

Shears which act in the positive direction of axes X, Y, and Z are considered to be positive. They are indicated by a plus sign at the upper right corner as: SM_{AX}^{+N}, SM_{AY}^{+N}, SM_{AZ}^{+N}. The magnitudes of the shears are indicated by the number N.

4. SMF_{AX}, SMF_{AY}, SMF_{AZ} are shear moment factors for points A, B, C, ... at which the shears act along axes X, Y, and Z respectively. An external load which acts in the positive direction of axes X, Y, and Z is considered to be positive. The nature of the sign of the external load is indicated by a plus or minus sign at the upper right corner as: $SMF_{AX}^{(+)}$, $SMF_{AY}^{(+)}$, $SMF_{AZ}^{(+)}$. As shown later the magnitude of the external load is, of course, taken as a unit.

B. Definitions and Constants

As the frames become more involved, it may be expected that new terms and constants should be introduced. For the convenience of the reader they are listed here.

1. <u>Definition of joint conditions</u>

Hinged = rotation permitted

Rigid = rotation prevented

Free = translation permitted

Fixed = translation prevented

These terms can be used in combination, e.g., Rigid-Fixed joint means the joint can neither rotate nor translate.

2. Stiffness factors

a. Stiffness factor for moments is the moment at the near end necessary to produce a unit rotation when the far end is Rigid-Fixed. For members with constant cross section the stiffness factor for moments is $\frac{4EI}{L}$. When the far end is Hinged-Fixed the stiffness factor for moments is $\frac{3EI}{L}$.

b. Stiffness factor for torques is the torque at the near end necessary to produce unit twist when the far end is Rigid-Fixed. For members with constant rectangular or square cross section the stiffness factor for torques is $\frac{k_1G(2a)^3\ (2b)}{L}$ (7). For other structural cross sections the reader should refer to the Bethlehem Steel Handbook (8).

c. Stiffness factor for shears is the shear necessary to produce unit deflection of one end with reference to the other. One end of the member is Rigid-Fixed; the other end may be Rigid-Free or Hinged-Free. For members with constant cross section, the stiffness factor for shear when the other end is Rigid-Free is $\frac{12EI}{L^3}$, while, when the other end is Hinged-Free it is $\frac{3EI}{L^3}$.

3. Distribution factors

a. Moment-torque distribution factor is the ratio of the stiffness factor for moment or torque to the sum of the stiffness factors for moments and torques on all the members meeting at the joint. As extreme cases, the distribution factor for moment or torque at the support with Rigid-Fixed condition can be considered as - 1.00 to the joint and zero to the member; while that at the support with Hinged-Fixed condition the distribution factor can be considered as zero to the joint and - 1.00 to the member. Moment-torque distribution factors have opposite signs from the unbalanced moments and torques.

b. The shear distribution factor for a member at a joint for shears which are parallel to the X or Y or Z axis of references taken through the joint is equal to the ratio of the shear stiffness factor in the member under consideration divided by the sum of the shear stiffness factors of all members adjacent and perpendicular to the line of action of the shear. The sign of the shear distribution factors is opposite to the sign of the unbalanced shears.

4. <u>Carry-over factors</u>

a. Carry-over factor for moments is the ratio of the induced moment at the far end ~~of~~ to the applied moment at the near end when the far end is in Rigid-Fixed condition. For constant cross section it is equal to +0.5. The plus sign is used because the carry-over moments have the same sign as the distributed moments.

b. Carry-over factor for torques is the ratio of the induced torque at the far end to the applied torque at the near end when the far end is in Rigid-Fixed condition. It is always equal to -1.0. The minus sign is used because the induced torque always has a sign opposite to that of the applied torque.

c. Carry-over factor for shears is the ratio of the induced shear at one end to the applied shear at the other end. It is equal to -1.0. The minus sign is used because the shears at both ends of a member are always equal and opposite in sign.

5. Shear moment factors

When a unit load is applied at a joint of a continuous space frame and in line with an axis of reference, the moments produced at the ends of all the members are called the shear moment factors. It is important to note that in the computation of the shear moment factors the joints are considered to be in the Rigid-Free condition, that is, they are permitted to translate but prevented from rotation.

For convenience, the author prefers to take the joints which lie in the top-front and top-right planes as the joints where the loads are applied.

C. Description of the Method

1. Shear distribution

When load P is applied, it is temporarily sustained by joint B, see Figure 2. The unbalanced shear at joint B can now be considered as equal and opposite to the external load P. The artificial restraint should, however, be released. In order to achieve this, the shear distribution method is developed.

Step 1. Consider joints A, D as Rigid-Fixed and joints B, C as Rigid-Free. Distribute the unbalanced shear to beams BA, CD and Columns BF, CG according to the shear distribution factors. The distributed shears at joint B have the same sign as the external load since the shear distribution factors have negative signs.

<u>Step 2.</u> Carry over the shears to ends A, D, F, G with signs changed since the carry-over factors for shears are minus one.

<u>Step 3.</u> The shears which have been carried over to Rigid-Fixed joints F, G need no further consideration. Joints A, D are, however, Rigid-Free instead of Rigid-Fixed as assumed. Therefore the shears on joints A, D must be redistributed to beams AB, DC and to columns AE, DH by treating joints B, C as Rigid-Fixed. The distributed shears have opposite signs to the carry-over shears on beams AB, DC from Step 2.

<u>Step 4.</u> Carry over the shears to ends B, C, E, H with signs changed.

<u>Step 5.</u> The shears which have been carried over to Rigid-Fixed joints E, H need no further consideration. The shears on joints B, C should be redistributed as outlined in Step 1.

<u>Step 6.</u> Repeat steps 1, 2, 3, 4 until the unbalanced shears on Rigid-Free joints A, B, C, D are negligible.

<u>Step 7.</u> Add algebraically the shears at the ends of beams AB, DC and columns AE, BF, CG, DH.

<u>Step 8.</u> Determine the moments on beams AB, DC and columns AE, BF, CG, DH corresponding to the shears as determined in Step 7. If the load P is unit, the moments determined here are called <u>Shear Moment Factors</u> $SMF_{BY}^{(-)}$. They are also fixed end moments for the unit load applied at joint B along Y-axis. The exponent has a minus sign because the load P is applied opposite to the positive direction of the Y-axis.

2. Moment-torque distribution

Step 1. Determine fixed end moments in the usual manner if the load is applied between the joints, or by shear distribution method if the load is applied at a joint. See Figures 2 and 4.

Step 2. Distribute the moments, considering the joints A, B, C, D as Hinged-Fixed. The moments at joints are now in equilibrium.

Step 3. Carry over the moments and torques.

Step 4. Add fixed-end moments, distributed moments, carry-over moments (no torques) for each member. For loads applied at joints, add only the distributed and carry-over moments for each member. Divide these summations by the length of each corresponding member. This operation gives the shear at each end of the member.

Step 5. Determine unbalanced shears at joints A, B, C along X and Y axes by adding the shears that were obtained in Step 4.

Step 6. Correct the unbalanced shears at joints A, B, C by using $SMF_{AY}(\pm)$, $SMF_{BY}(\pm)$, $SMF_{CX}(\pm)$, $SMF_{BX}(\pm)$ remembering the correction shears are opposite in sign to the unbalanced shears. The shears at joints are now in equilibrium. The joints A, B, C, D are Rigid-Free during this operation.

Step 7. Determine unbalanced moments at joints by adding carry-over moments, torques and shear moments.

Step 8. Distribute the moments as described in Step 2.

Step 9. Carry over the moments and torques.

Step 10. Add the distributed moments and carry-over moments. The shears at the ends of each member can be obtained as outlined in Step 4.

Repeat Steps 5, 6, 7, 8, 9 and 10 until the unbalanced moments and shears are negligible.

The algebraic sum of the moments and torques at the end of a member will give the desired final solution at that end. The final shears at each end of a member may be computed from the moments in the usual manner.

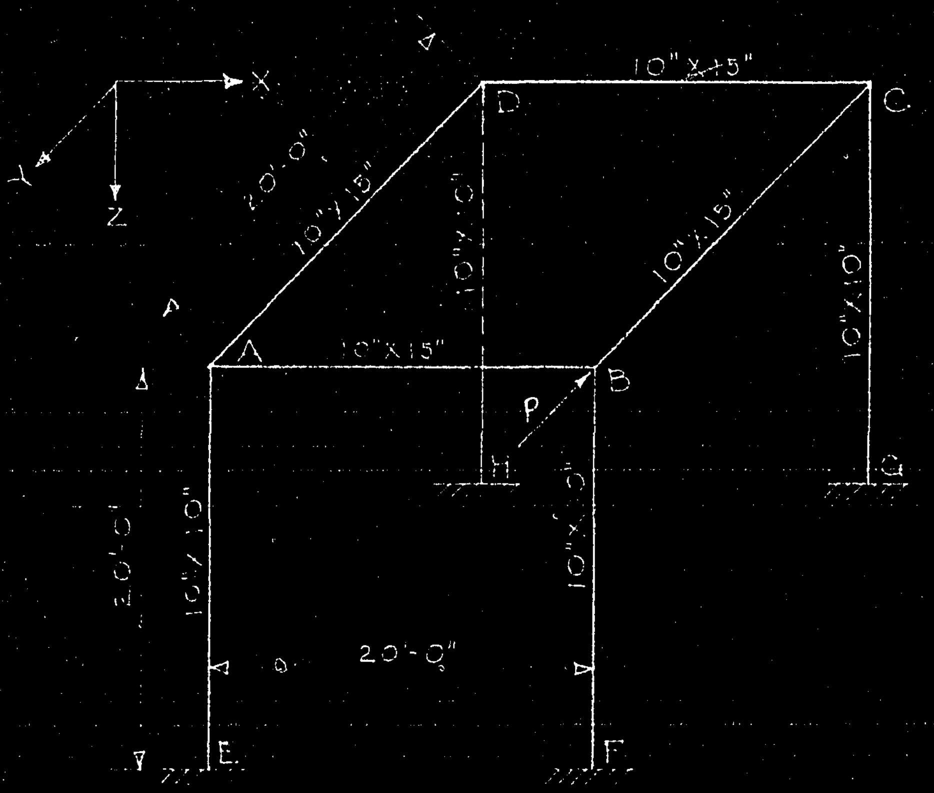
D. Typical Applications

To show how this method can be applied to practical problems, two frames, A and B, are chosen. On Frame A, six different loadings are applied. These six loading conditions are solved first without permitting sidesway. In order to correct for sidesway this frame is loaded with a corner load which will be designated in this thesis as a Master Load. The values of the shears, moments and torques developed by the master loading condition have been solved completely (see Figure 6).

On Frame B, a vertical load is applied (see Figure 18). This frame is also solved completely. The author wishes to describe the method of analysis used to obtain the solution of these frames in detail.

1. Frame A

The dimensions of this frame are shown in Figure 2. The moments of inertia for beams and columns with respect to the horizontal axes, X-axis and Y-axis, are shown in Figures 2 and 3. The stiffness factor for moments in beams is given as 46.8E. The stiffness factor for moments in columns is given as 13.9E. The stiffness factor for torques in beams



FRAME A

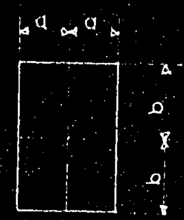
$I_x = I_y = \frac{10 \times \overline{15}^3}{12} = 2{,}810^{IN.^4}$ FOR BEAMS

$I_x = I_y = \frac{10^4}{12} = 833^{IN.^4}$ FOR COLUMNS

$I_z = \frac{15 \times \overline{10}^3}{12} = 1250^{IN.^4}$ FOR BEAMS

STIFFNESS FACTOR FOR ~~TORSION~~ Torques *

$$= \frac{k_1 G \, (2a)^3 (2b)}{l}$$



b/a	1.0	1.2	1.5	2.0	2.5	3	4	5	10	∞
k_1	0.1406	0.166	0.196	0.229	0.249	0.263	0.281	0.291	0.312	0.333

FIG. 2 FRAME A, DIMENSIONS

* TIMOSHENKO, S. "Theory of Elasticity", p. 248. McGRAW-HILL BOOK COMPANY INC., NEW YORK, 1934

MODULUS OF ELASTICITY = E

POISSON'S RATIO = 0.2

MODULUS OF ~~RIDGITY~~ Rigidity $= G = \dfrac{E}{2(1+0.2)} = \cancel{46.8E}$ $0.4167E$

$K = \dfrac{4EI_x}{L} = \dfrac{(4E)(2810)}{20 \times 12} = 46.8E$ FOR BEAMS (BENDING)

$K = \dfrac{4EI_x}{L} = \dfrac{(4E)(833)}{20 \times 12} = 13.9E$ FOR COLUMNS (BENDING)

$K = \dfrac{4EI_z}{L} = \dfrac{(4E)(1250)}{20 \times 12} = 20.833E$ FOR BEAM (BENDING)

$K^* = \dfrac{k_1 G (2a)^3 (2b)}{L} = \dfrac{0.196 \times 0.4167E \times (2 \times 5)^3 \times (2 \times 7.5)}{20 \times 12} = \dfrac{1225}{20 \times 12} = 5.1E$ BEAMS

$K^* = \dfrac{k_1 G \times 10^4}{L} = \dfrac{0.1406 \times 0.4167E \times 10^4}{20 \times 12} = 2.441E$ COLUMNS

ΣK'S $= 46.8 + 13.9 + 5.1 = 65.8$ FOR M_x & M_y

$$\left.\begin{aligned} D.F. &= \frac{46.8}{65.8} = 0.711 \\ D.F. &= \frac{13.9}{65.8} = 0.211 \\ D.F. &= \frac{5.1}{65.8} = 0.078 \end{aligned}\right] \text{ FOR } M_x \text{ \& } M_y$$

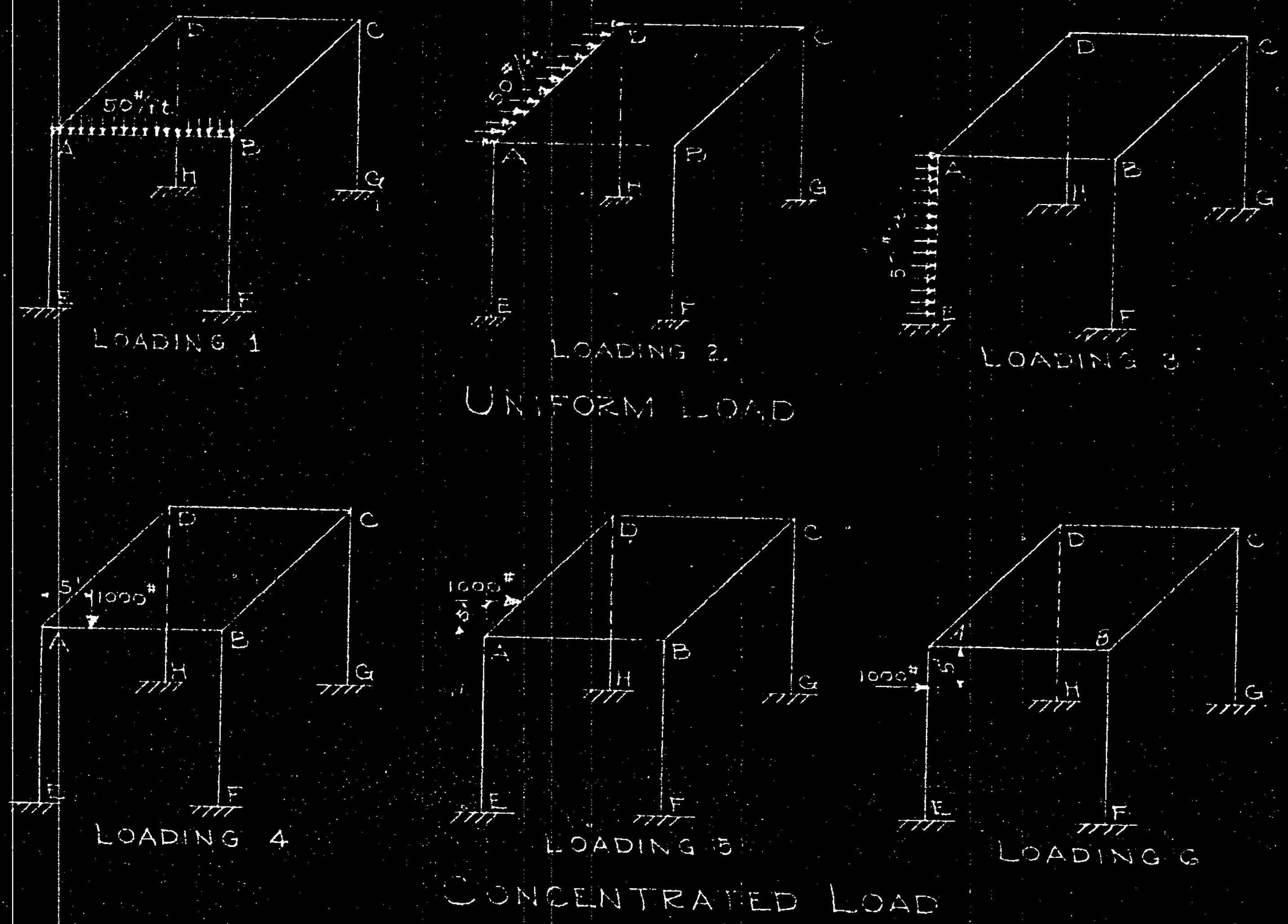
ΣK'S $= 20.833 + 20.833 + 2.441 = 44.107$ FOR M_z

$$\left.\begin{aligned} D.F. &= \frac{20.833}{44.107} = 0.472 \\ D.F. &= \frac{20.833}{44.107} = 0.472 \\ D.F. &= \frac{2.441}{44.107} = 0.056 \end{aligned}\right] \text{ FOR } M_z$$

FIG. 3 FRAME A, CONSTANTS

FIG. 4

LOADING CONDITIONS FOR FRAME A



is given as 5.1E. Regarding the formula for twisting moments of rectangular sections the reader should refer to the book entitled, "Theory of Elasticity" by S. Timoshenko, page 249. Now the sum of the stiffness factors is equal to 46.8 + 13.9 + 5.1 = 65.8. The distribution factors are therefore equal to 0.711, 0.211 and 0.078 for beams in bending, columns in bending and beams in torsion respectively. These distribution factors are used for the moments and torques in the planes perpendicular to either X-axis or Y-axis. The distribution factors for the moments and torques in the planes perpendicular to Z-axis ~~are~~ given as 0.472, 0.472 and 0.056. By definition the moments and torques in those planes are, of course, called M_x, M_y and M_z respectively.

For the time being six solutions are made by assuming that the joints are Hinged-Fixed so that no lateral displacements are allowed. There are three uniform loading conditions and three concentrated loading conditions as given in Figure 4. The computations are arranged in Tables 1 to 6, inclusive. Notice that in those tables the first row gives the name of the joints; the second row gives the kind of moments, whether they are M_x, M_y, or M_z; the third, fourth and fifth rows give the distribution factors, DF, and carry-over factors, COF. The fixed end moments are computed and entered in the tables with the proper signs attached to them.

Now the fixed end moment, FEM, for uniform load is equal to $WL^2/12$, and for concentrated load FEM = Pab^2/L^2 and Pa^2b/L^2 for the end near the load and the end farther away from the load, respectively. In the ordinary case, b is always greater than a. When b is equal to a, the load

TABLE 1. FRAME A. LOADING - 1, HINGED - FIXED

JOINT	A			B			C			D			E	F	G	H
	My			My			My			My			My	My	My	My
	AD*	AE	AB	BC*	BF	BA	CB*	CG	CD	DA*	DB	DC	EA	FB	GC	HD
	D.F.=0.078	D.F.=[illegible]	D.F.=0.311	D.F.=0.079	D.F.=[illegible]	D.F.=[illegible]	D.F.=[illegible]	D.F.=0.211	D.F.=0.311	D.F.=0.078	D.F.=0.211	D.F.=0.711				
	C.O.F.=-1.0	C.O.F.=+0.5	C.O.F.=+0.5	C.O.F.=-1.0	C.O.F.=+0.5	C.O.F.=+0.5	C.O.F.=-1.0	C.O.F.=+0.5	C.O.F.=+0.5	C.O.F.=-1.0	C.O.F.=+0.5	C.O.F.=-0.5				
F.E.M.			-1666.7			+1666.7										
1st DIS.	+33.[illegible]	+357.7	+1195.[illegible]	-130.[illegible]	-331.7	-1155.0										
C.O.			-582.[illegible]			-5[illegible]2.5	+130.0			-130.0			+175.[illegible]	-175.[illegible]		
2nd DIS.	+[illegible]	+125.[illegible]	+421.[illegible]	-42.2	-125.[illegible]	-421.2	-10.[illegible]	-27.4	-32.4	+10.2	+27.4	+32.4				
C.O.	-10.2		-210.7	+10.2		-210.7	+14.[illegible]		+46.[illegible]	-4.[illegible]		-46.2	+62.5	-62.5	-13.7	+13.7
3rd DIS.	+17.2	+45.[illegible]	+157.1	-17.2	-45.[illegible]	-157.1	-7.2	-16.5	-35.7	+7.2	+19.5	+35.7				
C.O.	-7.2		-78.5	+7.2		+78.5	+17.[illegible]		+32.[illegible]	-17.[illegible]		-32.[illegible]	+23.[illegible]	-23.[illegible]	-9.8	+9.8
4th DIS.	-6.7	+14.[illegible]	+54.[illegible]	-6.7	-18.[illegible]	-5[illegible]	+3.9	-10.[illegible]	-35.[illegible]	+3.9	+10.[illegible]	+35.[illegible]				
C.O.	-3.3		-32.5	+3.3		+32.5	+2.[illegible]		+17.5	-3.7		-17.5	+9.0	-9.0	-5.3	+5.3
5th DIS.	+[illegible]	+7.[illegible]	+24.[illegible]	-3.[illegible]	-7.8	-24.5	-1.3	-5.[illegible]	-17.4	+1.3	+5.2	+17.4				
C.O.	[illegible]		-12.2	+1.9		+12.2	-[illegible]		+8.7	-2.2		-8.7	+3.7	-3.7	-2.2	+2.2
6th DIS.	+1.1	+3.[illegible]	+10.[illegible]	-1.[illegible]	-3.[illegible]	-10.0	-0.3	-2.4	-8.[illegible]	+0.8	+2.4	+8.[illegible]				
C.O.	[illegible]		+5.[illegible]	+[illegible]		+5.[illegible]	+1.1		+4.0	-1.1		-4.0	+1.5	-1.5	-1.2	+1.2
7th DIS.	+2.5	+1.[illegible]	+4.3	-2.5	-1.2	-4.[illegible]	-0.4	-1.1	-3.[illegible]	+0.4	+1.1	+3.[illegible]				
C.O.	-0.4		-2.[illegible]	+0.4		-2.1	+0.3		-1.[illegible]	-0.[illegible]		-1.8	+0.8	-0.8	-0.5	+0.5
8th DIS.	+0.[illegible]	+1.5	+1.5	-0.[illegible]	-0.5	-1.2	-0.2	-0.[illegible]	-1.[illegible]	+0.2	+0.5	+1.6				
C.O.	-0.2		-0.8	+0.2		+0.3	+0.2		+0.3	-0.2		-0.3	-0.2	-0.2	-0.3	+0.3
9th DIS.	+0.[illegible]	+0.2	+0.[illegible]	-0.1	-0.2	-0.6	-0.1	-0.2	-0.7	+0.1	+0.2	+0.7				
C.O.	-0.1		-0.4	+0.1		+0.4	+0.[illegible]		+0.3	-0.1		+0.3	+0.1	-0.1	-0.1	+0.1
10th DIS.	+0.0	+0.1	+0.4	-0.0	-0.1	-0.4	-0.[illegible]	-0.1	-0.3	+0.0	+0.1	+0.3				
C.O.	-0.0		-0.[illegible]	+0.0		+0.2	+0.[illegible]		+0.2	-0.0		-0.2	+0.1	-0.1	-0.0	+0.0
11th DIS.	+0.0	+0.1	+0.1	-0.0	-0.1	-0.1	-0.[illegible]	-0.1	-0.1	+0.0	+0.1	+0.[illegible]				
TOTAL	+179.8	+55[illegible]	-73[illegible]	-179.8	-553.[illegible]	+723.[illegible]	+179.8	-67.1	-112.7	-179.8	+67.1	-112.7	+276.9	-276.9	-33.5	+33.5

TABLE 2. FRAME A, LOADING - 2, HINGED-FIXED.

JOINT	A			B			C			D			E	F	G	H
	Mz			Mz			Mz			Mz			Mz	Mz	Mz	Mz
	AD	AB*	AF	BC	BH*	BA	CB	CG*	CD	DA	DH*	DC	EA	FB	GC	HD
	DF=0.472	DF=0.056	DF=0.472	DF=0.472	DF=0.056	DF=0.472	DF=0.472	DF=0.056	DF=0.472	DF=0.472	DF=0.056	DF=0.472				
	COF=+0.5	COF=-1.0	COF=+0.5	COF=+0.5	COF=-1.0	COF=+0.5	COF=+0.5	COF=-1.0	COF=+0.5	COF=+0.5	COF=-1.0	COF=+0.5				
F.E.M.	+166.7									-166.7						
1st DIS.	-186.7	-93.3	-186.7							+186.7	+93.3	+166.7				
C.O.	+393.4					-393.4			+393.4	-393.4			+93.3			-93.3
2nd DIS.	-185.7	-22.0	-185.7	+185.7	+22.0	+185.7	-185.7	-22.0	-185.7	+185.7	+22.0	+185.7				
C.O.	+92.8		+92.8	-92.8		-92.8	+92.8		+92.8	-92.8		-92.8	+22.0	-22.0	+22.0	-22.0
3rd DIS.	-87.6	-10.4	-87.6	+87.6	+10.4	+87.6	-87.6	-10.4	-87.6	+87.6	+10.4	+87.6				
C.O.	+43.8		+43.8	-43.8		-43.8	+43.8		+43.8	-43.8		-43.8	+10.4	-10.4	+10.4	-10.4
4th DIS.	-41.3	-5.0	-41.3	+41.3	+5.0	+41.3	-41.3	-5.0	-41.3	+41.3	+5.0	+41.3				
C.O.	+20.7		+20.7	-20.7		-20.7	+20.7		+20.7	-20.7		-20.7	+5.0	-5.0	+5.0	-5.0
5th DIS.	-19.6	-2.2	-19.6	+19.6	+2.2	+19.6	-19.6	-2.2	-19.6	+19.6	+2.2	+19.6				
C.O.	+9.8		+9.8	-9.8		-9.8	+9.8		+9.8	-9.8		-9.8	+2.2	-2.2	+2.2	-2.2
6th DIS.	-9.3	-1.0	-9.3	+9.3	+1.0	+9.3	-9.3	-1.0	-9.3	+9.3	+1.0	+9.3				
C.O.	+4.6		+4.6	-4.6		-4.6	+4.6		+4.6	-4.6		-4.6	+1.0	-1.0	+1.0	-1.0
7th DIS.	-4.3	-0.6	-4.3	+4.3	+0.6	+4.3	-4.3	-0.6	-4.3	+4.3	+0.6	+4.3				
C.O.	+2.2		+2.2	-2.2		-2.2	+2.2		+2.2	-2.2		-2.2	+0.6	-0.6	+0.6	-0.6
8th DIS.	-2.1	-0.2	-2.1	+2.1	+0.2	+2.1	-2.1	-0.2	-2.1	+2.1	+0.2	+2.1				
C.O.	+1.0		+1.0	-1.0		-1.0	+1.0		+1.0	-1.0		-1.0	+0.2	-0.2	+0.2	-0.2
9th DIS.	-0.9	-0.2	-0.9	+0.9	+0.2	+0.9	-0.9	-0.2	0.9	+0.9	+0.2	+0.9				
C.O.	+0.5		+0.5	-0.5		-0.5	+0.5		+0.5	-0.5		-0.5	+0.2	-0.2	+0.2	-0.2
10th DIS.	-0.5	-0.0	-0.5	+0.5	+0.0	+0.5	-0.5	-0.0	-0.5	+0.5	+0.0	+0.5				
C.O.	+0.2		+0.2	-0.2		-0.2	+0.2		+0.2	-0.2		-0.2	+0.0	-0.0	+0.0	-0.0
11th DIS.	-0.2	-0.0	-0.2	+0.2	+0.0	+0.2	-0.2	-0.0	-0.2	+0.2	+0.0	+0.2				
TOTAL	+1027.5	-154.9	-962.6	[illegible]	+41.6	-217.5	[illegible]	-41.6	+217.5	-1027.5	+154.9	+962.6	+154.9	-41.6	+41.6	-154.9

TABLE 3. FRAME A, LOADING-3, HINGED-FIXED

TABLE 4 FRAME A, LOADING 4, HINGED-FIXED

JOINT	A			B			C			D			[illegible]	[illegible]	[illegible]	[illegible]
	AD*	A[illegible]	AB	BC*	B[illegible]	BA	CB*	C[illegible]	CD	DA*	D[illegible]	DC	[illegible]	[illegible]	[illegible]	[illegible]
	DF=0.075	DF=0.2[illegible]	DF=0.71[illegible]	DF=0.075	DF=0.21[illegible]	DF=[illegible]	DF=[illegible]	DF=[illegible]	DF=0.71[illegible]	DF=0.075	DF=[illegible]	DF=0.71[illegible]				
	C.O.F.=[illegible]	C.O.F.=0.5	C.O.F.=0.5	C.O.F.=[illegible]	C.O.F.=0.5	C.O.F.=0.5	C.O.F.=[illegible]	C.O.F.=0.5	C.O.F.=0.5	C.O.F.=[illegible]	C.O.F.=0.5	C.O.F.=0.5				
F.E.M.			-[illegible]			+937.5										
1st DIS.	+219.[illegible]	+533.[illegible]	+999.7	-78.[illegible]	-197.8	-[illegible]										
CO			-333.[illegible]			+903.8	+7[illegible]			-[illegible]			+[illegible]	-[illegible]		
2nd DIS.	+[illegible]	+70.2	+22[illegible]	-77.3	-211.0	-715.9	[illegible]	-[illegible]	-52.0	[illegible]	+[illegible]	+[illegible]				
CO	-17.[illegible]		-355.5	+[illegible].7		+[illegible]	+77.9		+[illegible]	-[illegible]		[illegible]	+[illegible]	-[illegible]	-[illegible]	+[illegible]
3rd DIS.	+2[illegible]	+75.6	+261.9	-9.7	-26.2	-96.2	-12.2	-2[illegible]	[illegible]	+4.[illegible]	+[illegible]	+[illegible]				
CO	-4.[illegible]		-44.[illegible]	+2.2		+[illegible]	+9.7		+[illegible]	-[illegible]		+61.4	+[illegible]	-[illegible]	[illegible]	-5.[illegible]
4th DIS.	+[illegible]	+10.1	+31.[illegible]	-11.2	-30.5	-102.9	[illegible]	[illegible]	-[illegible]	+[illegible]	+14.8	[illegible]				
CO	-6.5		-[illegible]	+2.[illegible]		+17.[illegible]	+1.[illegible]		+[illegible]	-3.8		[illegible]	[illegible]	-[illegible]	-3.[illegible]	+6.8
5th DIS.	+4.5	+12.5	+41.[illegible]	[illegible]	-4.[illegible]	-13.8	3.[illegible]	-[illegible]	+20.4	+1.1	+2.[illegible]	+[illegible]				
CO	-1.1		-[illegible]	+[illegible]		+20.8	+1.8		[illegible]	-[illegible]		-14.7	+[illegible]	-2.0	-1.[illegible]	+[illegible]
6th DIS.	+0.[illegible]	+1.7	+[illegible]	-1.[illegible]	-[illegible]	-17.0	-[illegible]	-1.1	-1.[illegible]	+1.[illegible]	+4.[illegible]	+15.[illegible]				
	-1.6		-2.[illegible]	+0.[illegible]		+2.8	+1.[illegible]		+1.8	-[illegible]		-2.[illegible]	+0.8	-2.5	-[illegible]	+2.[illegible]
7th DIS.	+0.8	+2.[illegible]	+2.[illegible]	-0.[illegible]	-[illegible]	-2.2	+0.7	-1.8	-[illegible]	+[illegible]	+[illegible]	+[illegible]				
CO	-0.2		-1.1	+0.7		+[illegible]	+0.3		[illegible]	-[illegible]		-[illegible]	+[illegible]	-[illegible]	-0.0	+0.[illegible]
8th DIS.	+0.1	+0.[illegible]	+0.9	-0.8	-0.9	-3.1	-0.1	-[illegible]	-1.[illegible]	+2.[illegible]	+0.8	+2.[illegible]				
CO	-0.[illegible]		-1.8	+0.1		+0.[illegible]	-0.[illegible]		+1.4	-0.1		[illegible]	+0.1	-0.5	-0.[illegible]	+0.4
9th DIS.	+0.1	+0.4	+1.4	-0.1	-0.1	-0.4	-0.1	-[illegible]	[illegible]1.2	+0.1	+[illegible]	+0.4				
CO	-0.1		-0.2	+0.1		+[illegible]	+0.1		+0.2	-0.1		+0.6	[illegible]	+0.0	-0.2	+0.1
10th DIS.	+0.0	+0.1	+0.2	-0.3	-0.2	-0.6	-0.[illegible]	[illegible]	-0.2	+0.1	+[illegible]	+0.[illegible]				
CO	-0.1		-0.[illegible]	+0.0		+0.1	-0.0		+0.2	-0.[illegible]		+0.[illegible]	+0.[illegible]	+0.1	-0.[illegible]	+0.[illegible]
11th DIS.	+0.0	+0.1	+0.[illegible]	-0.0	-0.0	+0.1	-0.0	-0.[illegible]	-0.1	+[illegible]	[illegible]	+0.1				
CO	-0.0		-0.0	+0.0		+0.1	+0.0		+0.1	-0.0		-0.[illegible]	+0.[illegible]	-0.0	+0.[illegible]	+0.[illegible]
12th DIS.	+0.0	+0.0	+0.0	-0.0	-0.0	-0.1	-0.0	-0.0	-0.1	+0.0	+0.0	+0.0				
TOTAL	+253.6	+769.4	-1022.7	-151.5	-478.4	+628.2	+11[illegible]	-67.[illegible]	-84.[illegible]	-253.[illegible]	+[illegible]	[illegible]	+284.7	-238.2	-[illegible]	+41.9

TABLE 5. FRAME A, LOADING - 5, HINGED - FIXED

JOINT	A			B			C			D			E	F	G	H
	M_z			M_z			M_z			M_z			M_z	M_z	M_z	M_z
	AD	AE*	AB	BC	BF*	BA	CB	CG*	CD	DC	DH*	DA	EA	FB	GC	HD
	[illegible]	[illegible]	DF=-0.472	DF=-0.472	DF=-0.056	DF=-0.472	[illegible]	DF=-0.056	[illegible]	[illegible]	DF=-0.056	DF=-0.472				
	[illegible]	C.O.F.=-1.0	C.O.F.=+0.5	C.O.F.=+0.5	C.O.F.=-1.0	C.O.F.=+0.5	C.O.F.=+0.5	C.O.F.=-1.0	C.O.F.=+0.5	C.O.F.=+0.5	C.O.F.=-1.0	C.O.F.=+0.5				
F.E.M.	+1312.5									-937.5						
1st DIS.	-1327.5	-157.5	-1327.5							+442.5	+52.5	+442.5				
C.O.	+221.3					-663.8			+221.3	-663.8			+157.5			-52.5
2nd DIS.	-104.5	-12.3	-104.5	+312.2	+37.2	+312.2	-104.5	-12.3	-104.5	+312.2	+37.2	+312.2				
C.O.	+156.7		+156.7	-52.2		-52.2	+156.7		+156.7	-52.2		-52.2	+12.3	-37.2	+12.3	-37.2
3rd DIS.	-147.9	-17.6	-147.9	+49.3	+5.8	+49.3	-147.9	-17.6	-147.9	+49.3	+5.8	+49.3				
C.O.	+24.6		+24.6	-74.0		-74.0	+24.6		+24.6	-74.0		-74.0	+17.6	-5.8	+17.6	-5.8
4th DIS.	-23.2	-2.8	-23.2	+69.0	+8.2	+69.5	-23.2	-2.8	-23.2	+69.0	+8.2	+69.5				
C.O.	+35.0		+35.0	-11.6		-11.6	+35.0		+35.0	-11.6		-11.6	+2.8	-8.2	+2.8	-8.2
5th DIS.	-33.0	-4.0	-33.0	+11.0	+1.2	+11.0	-33.0	-4.0	-33.0	+11.0	+1.2	+11.0				
C.O.	+5.5		+5.5	-16.5		-16.5	+5.5		+5.5	-16.5		-16.5	+4.0	-1.2	+4.0	-1.2
6th DIS.	-5.2	-0.6	-5.2	+15.6	+1.8	+15.6	-5.2	-0.6	-5.2	+15.6	+1.8	+15.6				
C.O.	+7.8		+7.8	-2.6		-2.6	+7.8		+7.8	-2.6		-2.6	+0.6	-1.8	+0.6	-1.8
7th DIS.	-7.4	-0.8	-7.4	+2.4	+0.4	+2.4	-7.4	-0.8	-7.4	+2.4	+0.4	+2.4				
C.O.	+1.2		+1.2	-3.7		-3.7	+1.2		+1.2	-3.7		-3.7	+0.8	-0.4	+0.8	-0.4
8th DIS.	-1.1	-0.2	-1.1	+3.5	+0.4	+3.5	-1.1	-0.2	-1.1	+3.5	+0.4	+3.5				
C.O.	+1.7		+1.7	-0.5		-0.5	+1.7		+1.7	-0.5		-0.5	+0.2	-0.4	+0.2	-0.4
9th DIS.	-1.6	-0.2	-1.6	+0.5	+0.0	+0.5	-1.6	-0.2	-1.6	+0.5	+0.0	+0.5				
C.O.	+0.3		+0.3	-0.8		-0.8	+0.3		+0.3	-0.8		+0.8	+0.2	-0.0	+0.2	-0.0
10th DIS.	-0.3	-0.0	-0.3	+0.8	+0.0	+0.8	-0.3	-0.0	-0.3	-0.8	+0.0	+0.2				
C.O.	+0.4		+0.4	-0.2		-0.2	+0.4		+0.1	-0.2		-0.2	+0.0	-0.0	+0.0	-0.0
11th DIS.	-0.4	-0.0	-0.4	+0.2	+0.0	+0.2	-0.4	-0.0	-0.4	+0.2	+0.0	+0.2				
C.O.	+0.1		+0.1	-0.2		-0.2	+0.1		+0.1	-0.2		-0.2	+0.0	-0.0	+0.0	-0.0
12th DIS.	-0.1	-0.0	-0.1	+0.2	+0.0	+0.2	-0.1	-0.0	+0.1	+0.2	+0.0	+0.2				
C.O.	+0.1		+0.1	-0.0		-0.0	+0.1		+0.1	-0.0		-0.0	+0.0	-0.0	+0.0	-0.0
13th DIS.	-0.1	-0.0	-0.1	+0.0	+0.0	+0.0	-0.1	-0.0	-0.1	+0.0	+0.0	+0.0				
TOTAL	+1064.9	-196.0	-1463.5	+304.4	+55.0	-359.4	-91.4	-35.5	+126.9	-854.4	+107.5	+746.9	+196.0	-55.0	+35.5	-107.5

TABLE 6 FRAME A. LOADING G-6, HINGED-FIXED

is at the center of the member, the fixed end moments for both ends will then be the same. For the problems in hand the fixed end moments for uniform load equal 50 x 20 x 20/12 = 1666.7 ft. lb. The moments are the same at both ends of the member but with different signs. The fixed end moment for the concentrated load at the near end equals 1000 x 5 x 15 x 15/20 x 20 = 2812.5 ft. lb. The fixed end moment for the concentrated load at the farther end equals 1000 x 5 x 5 x 15/20 x 20 = 937.5 ft. lb. Be sure to enter them with the proper sign. The distribution process is applied exactly for planar frames with sidesway prevented.

In order to hold the structure against the lateral displacements one does not need to make every joint fixed. For the problem in hand if one could fix the joints A and B against translation along the direction of Y-axis and the joints C and B against translation along the direction of X-axis, the sidesway of the whole frame would be prevented.

Actually, however, the structure has to have the lateral displacements if the external load and the elastic behavior of the structure call for it. Therefore, the restraints set up by the Hinged-Fixed joints should be released by the application of equal and opposite corner loads. The problem of correction for sidesway is then reduced to the problem of making the solutions with corner loads at each of the Hinged-Fixed joints. Fortunately, for the frame considered one such solution is all that is needed, for the solutions with corner load applied at other joints can be obtained by symmetry. One can also use the actual shears at all the joints as the applied loads, the correction is then made by one such solution alone.

Before one attempts a solution of this kind a concept of prime importance has to be introduced. As one knows a load applied at a joint in continuous space frames influence not only the members in the panel but also the members in the other panels as well. This influence must be known before any attempt is made for the correction of the shears.

The device known as the Shear Moment Factors is now described in detail. In Figure 5 a concentrated load of 1000 pounds is applied at B opposite to the direction of Y-axis. For the time being joints A and D are assumed as Rigid-Fixed. Then joints B and C are assumed to be in a Rigid-Free condition, and the moment induced by the load in this case will be taken up by beams AB, DC, and columns BF, CG according to the ratio of I/L^2 of each member to the sum of I/L^2, values for all the involved members.

In case the length of the beams is different from the length of the columns the shears should first be distributed according to their I/L^3 ratios. Then the moments at the ends of a member will be equal to the corresponding shear multiplied by the length of the member. Half of this moment will be entered at each end of the member. The shear distribution factors will be equal to $12I/L^3$ for the members rigid at both ends and $3I/L^3$ for the members rigid at one end and hinged at the other. The moment on the member, which is rigid at both ends, will then be equal to the appropriate shear multiplied by its own length, half of which will go to each end of the member. The moment on the member, which is rigid at one end and hinged at the other will be equal to the appropriate shear multiplied by its own length. This moment will be entered at the rigid end alone.

Now it is the time to come back to the original problem. As shown in Figure 5 the distribution factor for columns BF and CG is 0.19998 and the distribution factor for beams BA and CD is 0.30002. The corner load thousand pounds will give 1000 x 20 = 20,000 ft. lb. Of that 3000.2 is distributed to each end of the beams BA and CD and 1999.8 is distributed to each end of the columns BF and CG. The moments distributed to the beams BA and CD are called M_z and the moments distributed to the columns BF and CG are called M_x. One should notice that all of them bear the negative sign because they tend to rotate the joints counterclockwise.

Now since there is nothing to hold the joints A and D, a displacement must take place caused by the shears at these joints. However, as discussed before, the joint D will be fixed if the joint A is fixed, therefore the shears can be considered as applied to the joint A alone. The shear at joint A is then found by dividing the sum of the moments on beams AB and DC by the length of the beam, i.e., (3000.2 x 4)/20 = 600.04. Now the joints B and C are assumed this time as Rigid-Fixed at the new position. Meanwhile the joints A and D are assumed in the Rigid-Free condition. The moment due to the shear at joint A is equal to 600.04 x 20 = 12,000.8 ft. lb. Of that 1800.3 is distributed to each end of the beams AB and DC and 1199.9 is distributed to each end of the columns AE and DH. Notice that the moments on beams AB and DC bear the positive sign because they tend to rotate the joints clockwise. These new moments on beams AB and DC will set up new shear at joint E. This shear should be released exactly as before. This process is repeated as often as the accuracy requires until the restraint on the joints is negligible. The final results are obtained by adding algebraically all the moments at each end of each member.

These results give the moments induced by the corner load when the structure is permitted to deflect to its natural position with the rotation of the joints prevented. This process is just opposite to the distribution process, which permits the rotation but prevents the translation of all the joints.

By dividing the results by 1000 one obtains the shear moment factors for a unit load at joint B and acting along the Y-axis. The shear moment factors for joint A along the Y-axis and the shear moment factors for joints C and B along the X-axis are obtained by symmetry. They are shown as the S.M.F. values in the first four rows of Table 7. One should notice that the directions of all the corner loads are opposite to the directions of the respective axes. For convenience they are called negative.

As discussed before, the key for the correction of sidesway is to make a complete solution with corner loads applied at certain joints. At joint B of Frame A a corner load of 1000 lb. is applied opposite to the direction of the Y-axis. The computations giving a complete solution for this condition of loading on Frame A are designated as Master Load and are arranged in Table 7. The stiffness factors for moments and torques and the distribution factors are obtained as before. The fixed end moments are equal to the shear moment factors for joint B along the Y-axis multiplied by 1000. As one may notice these moments give perfect balance so far as the shears are concerned but the moments are not in equilibrium at the joints. The distribution process can now be performed and the

MOMENTS IN FT-LBS.

* INDICATES ~~TORSION~~ Torques

FIG. 6 MASTER-LOAD SOLUTION FOR FRAME A

moments and torques can then be carried over to the other end. This distribution process coupled by the carry-over process will create the restraints which the joints offered to the frame. In order to release these restraints equal and opposite shears have to be applied to each of the Hinged-Fixed joints.

In order to find the shears at the joints, the moments in the horizontal and vertical members have to be considered. For clearness in following the computations they are rearranged from the first distribution of moments in Table 7 as follows:

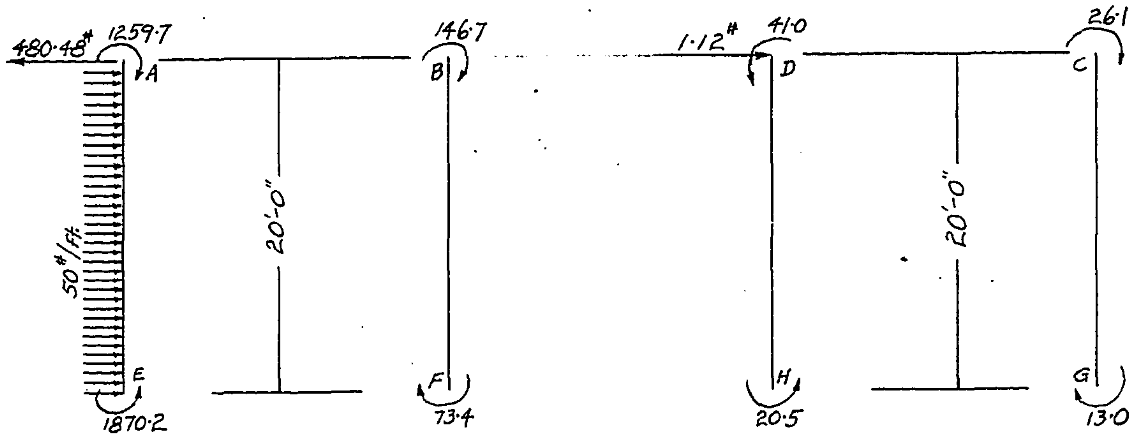
(1)	(2)	(3)	(4)	(5)	(6)
Col. AE	Col. BF	Col. CG	Col. BF	Beam AB	Beam CB
Col. DH	Col. CG	Col. DH	Col. AE	Beam DC	Beam DA
M_x	M_x	M_y	M_y	M_z	M_z
+ 395.6	+ 659.4	0.0	0.0	+ 885.0	+ 885.0
+ 197.8	+ 329.7			+ 442.5	+ 442.5
+ 395.6	+ 659.4	s = 0	s = 0	+ 885.0	+ 885.0
+ 197.8	+ 329.7			+ 442.5	+ 442.5
+1186.8	+1978.2			+ 885.0	+ 885.0
				+ 442.5	+ 442.5
s=59.34	s=98.91			+ 885.0	+ 885.0
				+ 442.5	+ 442.5
				+5310.0	+5310.0
				s= 265.5	s= 265.5

Item (1) gives a positive shear, +59.34 at joint A along the Y-axis; (2) gives a positive shear, +98.91 at joint B along the Y-axis; (3) and (4) give no shears along the X-axis since they are zero; (5) gives negative shear, -265.5 at joint A and positive shear, +265.5 at joint B along the Y-axis; (6) gives positive shear, +265.5 at joint C and negative shear, -265.5 at joint B along the X-axis. Adding algebraically, the shear at joint A along the Y-axis equals -265.5 + 59.34 equals -206.16; the shear at joint B along the Y-axis equals +265.5 + 98.91 equals +364.41; the shear at joint C along the X-axis equals +265.5 + 0 equals +265.5; the shear at joint B along the X-axis equals 265.5 + 0 equals -265.5. The correction shears will, of course, be equal and opposite to these. In Table 7 one notices that at the left margin of every sheet, after each distribution and carry-over process there are four rows marked as $SM_{AY}^{+206.16}$, etc. The letters SM stand for shear moment; the subscript AY indicates that the correction shear is applied at joint A along the Y-axis; the exponent +206.16 indicates that the amount of the shear is 206.16 and is positive in its direction.

After the unbalanced shears are corrected, the distribution process should be repeated. The unbalanced moment at a joint will be equal to the algebraic sum of the carry-over moments and the moments due to the correction shears. The alternate balancing of moments and shears should be repeated until the unbalanced shears and moments are negligible. The final results are shown in Table 7. The solutions for the other corner loads are obtained by symmetry. Dividing these results by 1000, the correction factors, CF for unit shear at different joints are obtained.

In Tables 8 and 9, one notices that there are four correction factors, CF; their subscripts and exponents have the same meaning as described in the last paragraph. For each loading condition, the moments obtained by considering the joints as Hinged-Fixed, are recorded at the top. The corrections, Corr, are made by the use of the correction factors, CF. It may be noticed that if the correction shear has the same sign as that of the correction factors, the signs will remain the same, otherwise they should all be reversed.

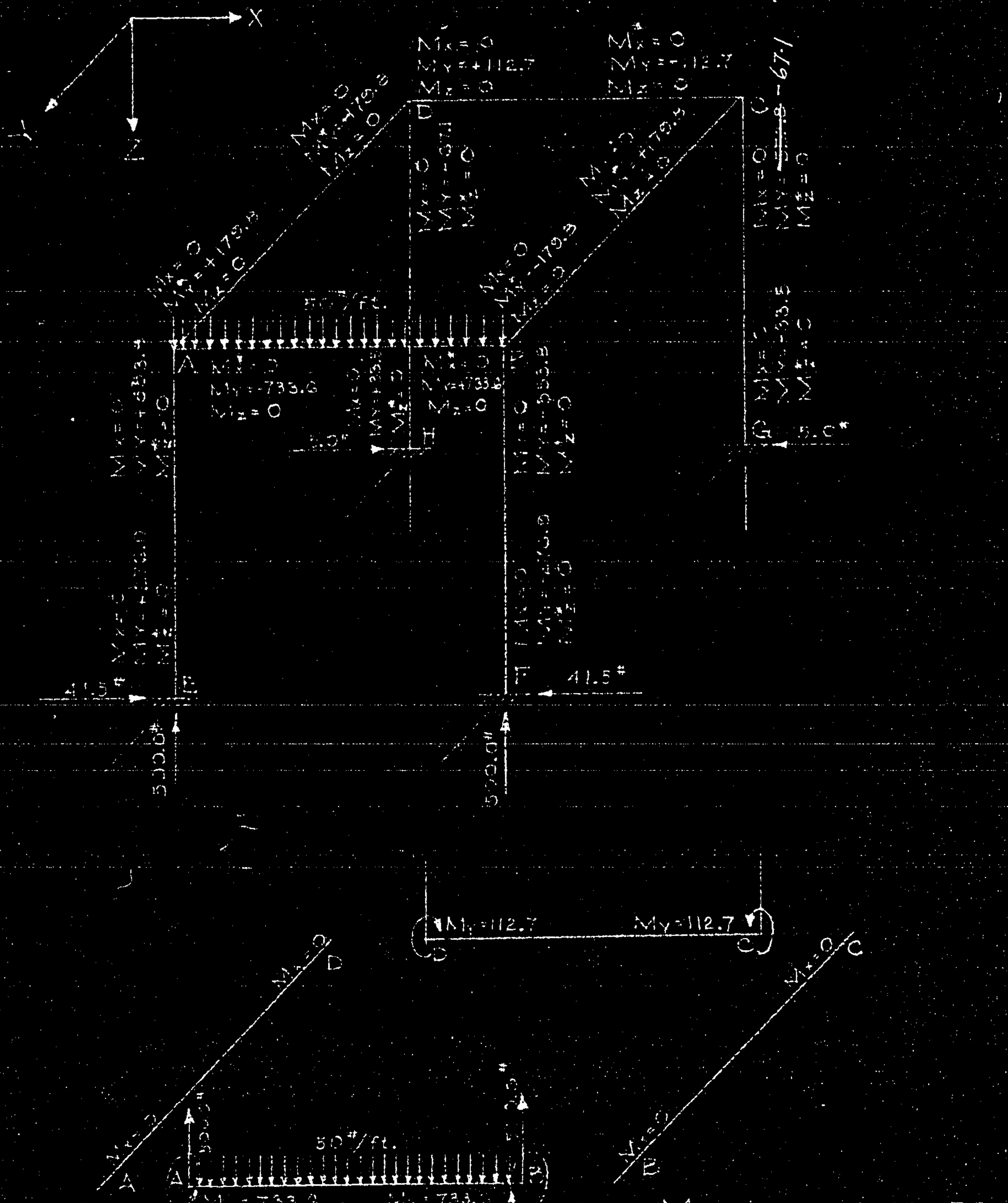
The restraint of the joints can be obtained as outlined before. Taking for instance the Loading 3 for uniform load in Table 8, the moments on the members are shown as follows:



The restraints expressed as shears which the joints must have offered to the frame are indicated in the above figure. The correction shears are recorded at the left side of each sheet in Tables 8 and 9. The final results for the six loadings are recorded on the frames in Figures 7 to 12, inclusive. The reactions are given for each case. It can be shown that all of them satisfy the six equations for the equilibrium of the forces in space.

2. Frame B, vertical load on intermediate cross beam

This frame is similar to Frame A except that a cross beam QR is added. The general dimensions and the position of a vertical load of 1000 pounds on beam QR are shown in Figure 13. One should notice that the frame belongs to the classification, Three Directions of Displacement, as shown in Figure 1. Frame B is typical of the most general arrangement that one may meet in the solution of the continuous space frame in which the members are at right angles to each other.



MOMENTS IN FT.-LBS.
* INDICATES Torques ~~TORSION~~

FIG. 7 SOLUTION FOR LOADING- I

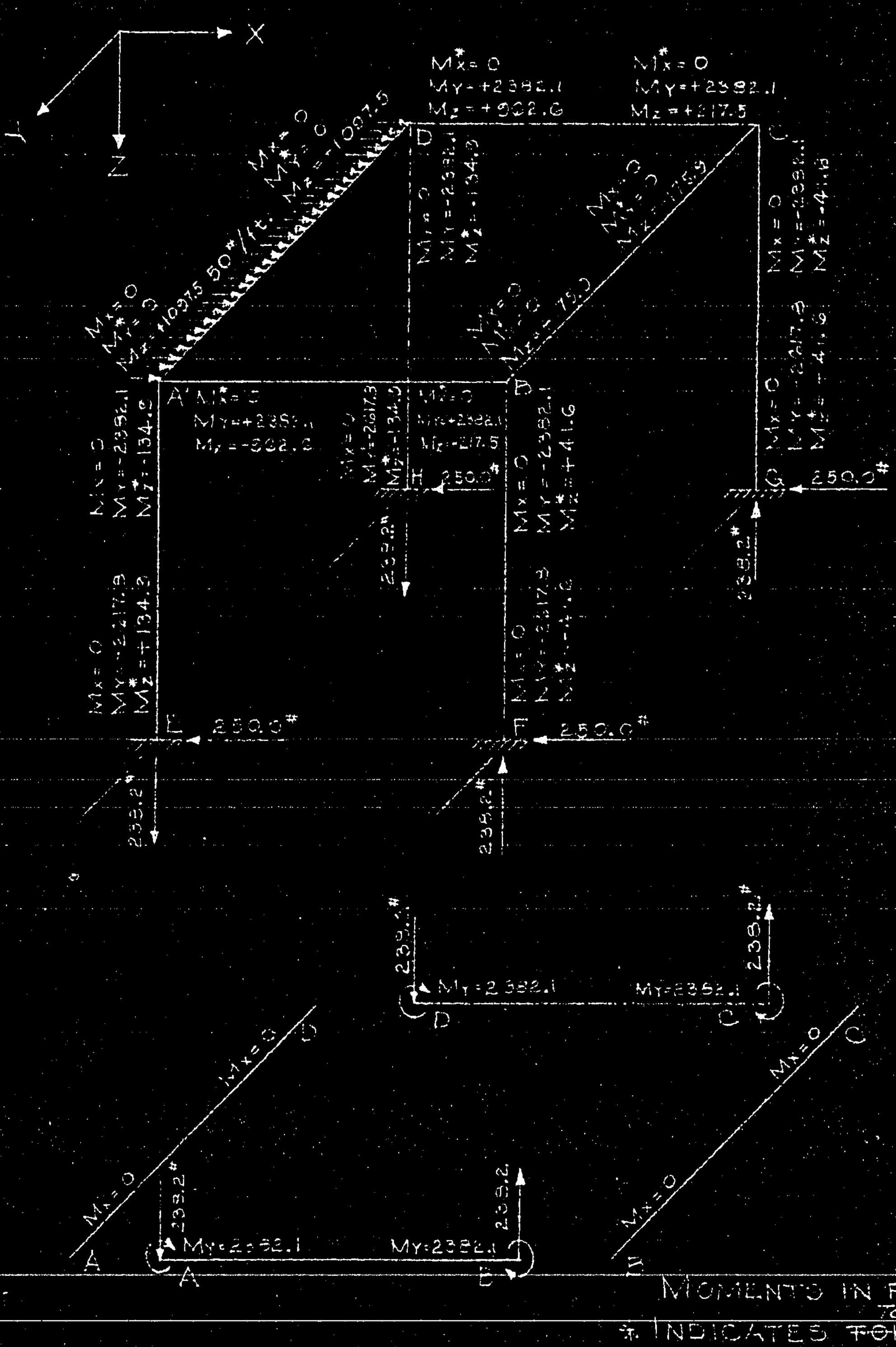


FIG 8 SOLUTION FOR LOADING-2

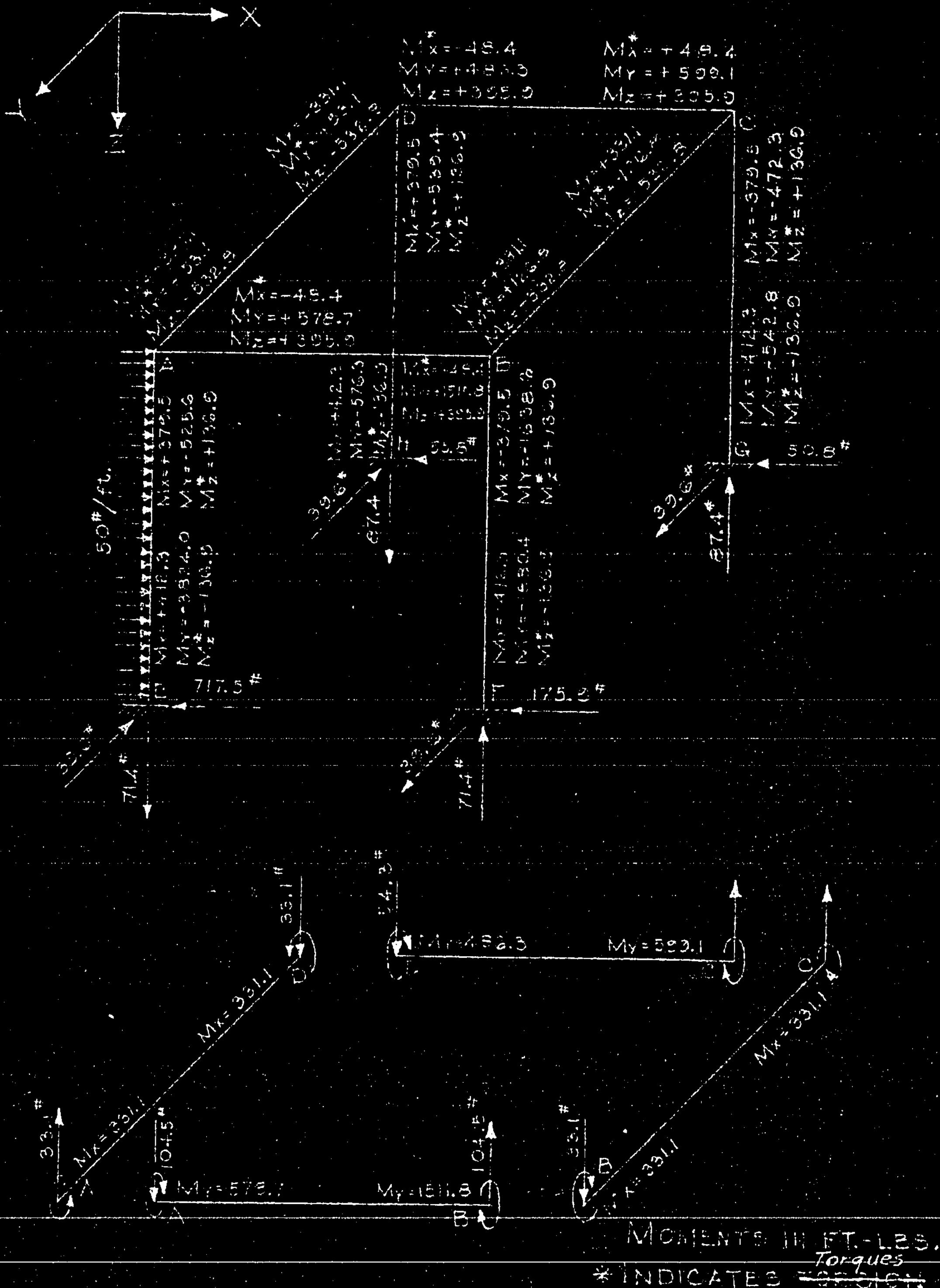
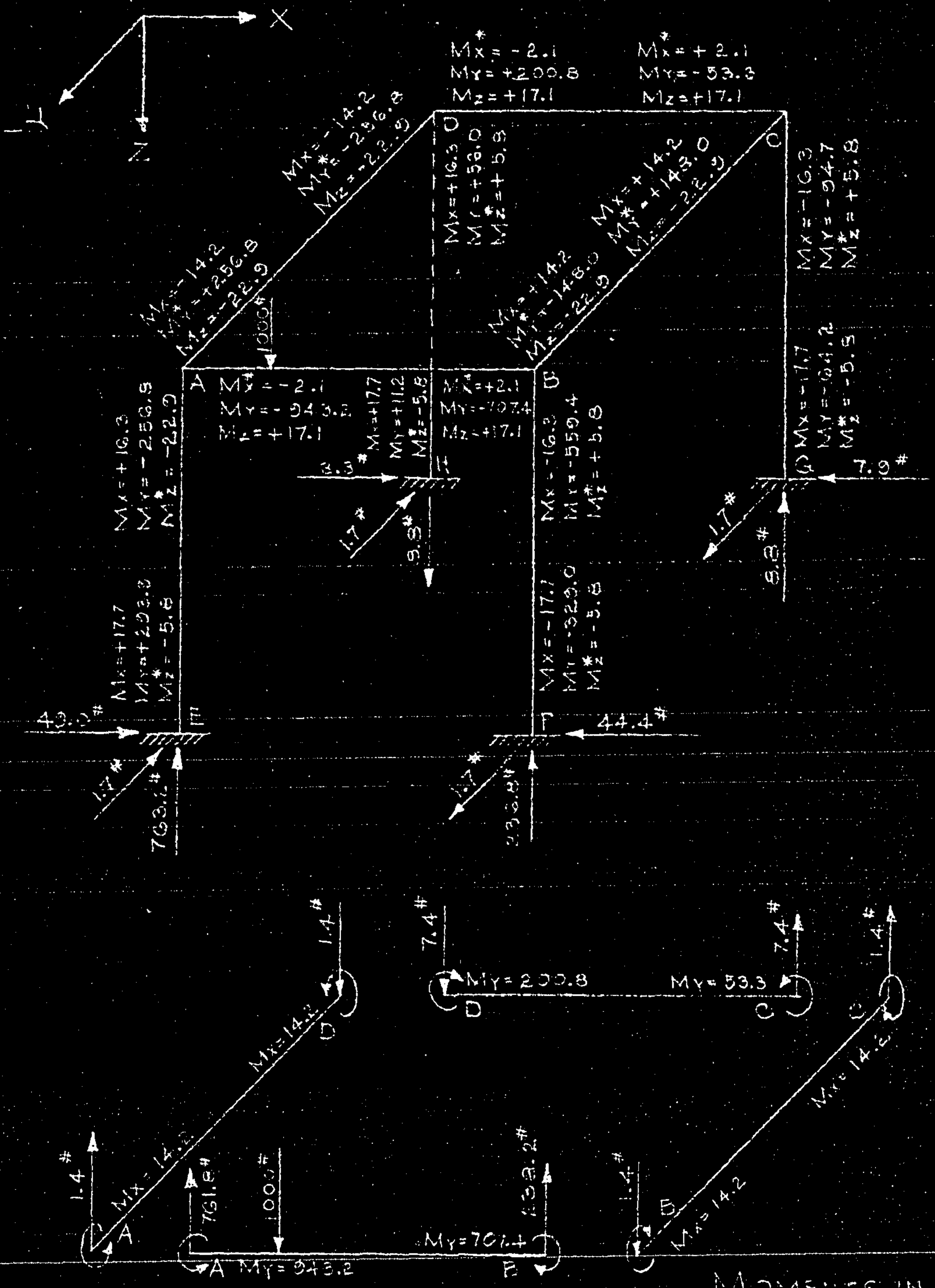


FIG. 9 SOLUTION FOR LOADING - 3.



MOMENTS IN FT-LBS
* INDICATES ~~TORSION~~ Torques

FIG. 10 SOLUTION FOR LOADING - 4

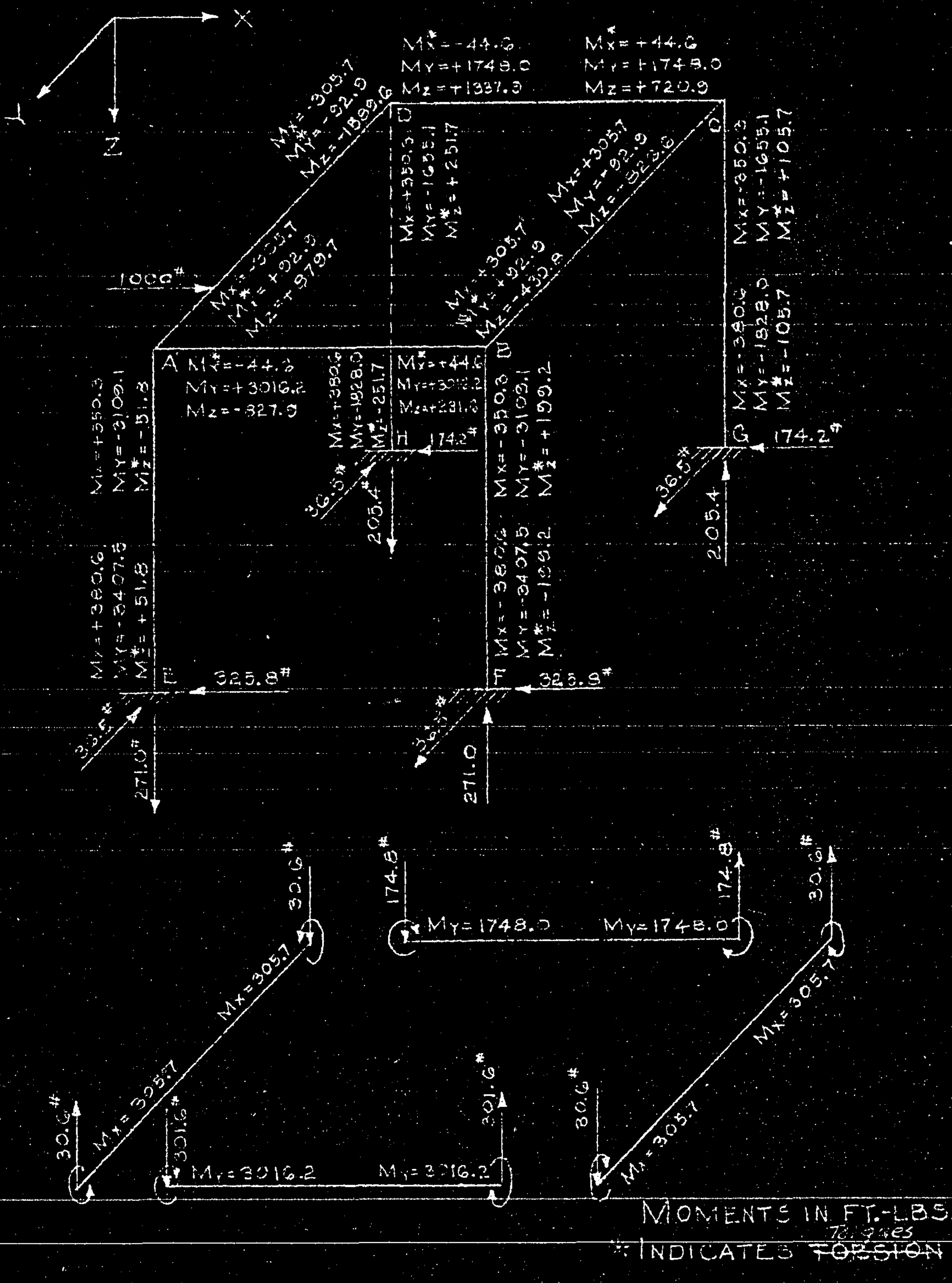


FIG 11, SOLUTION FOR LOADING-5

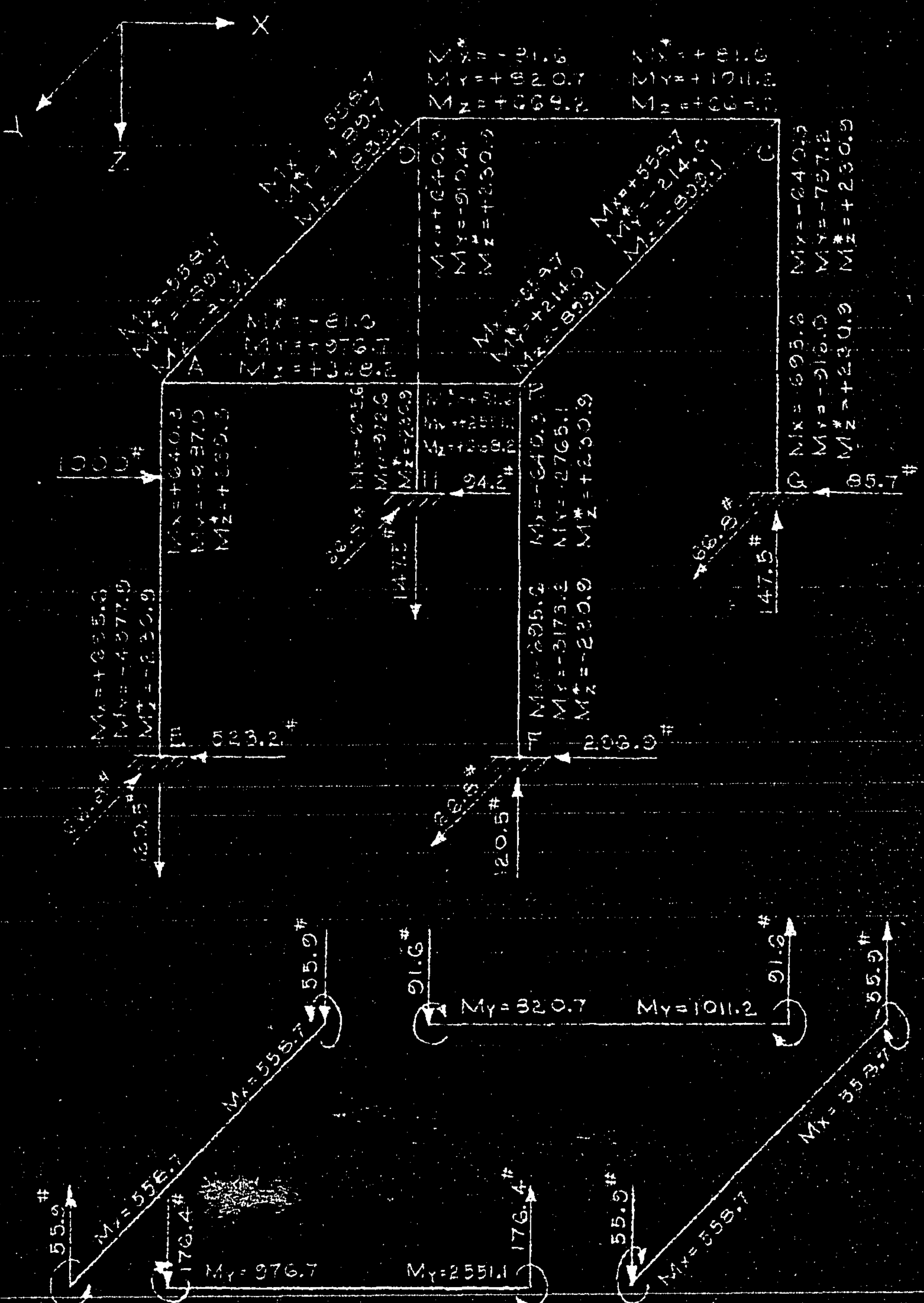


FIG. 12 SOLUTION FOR LOADING-6

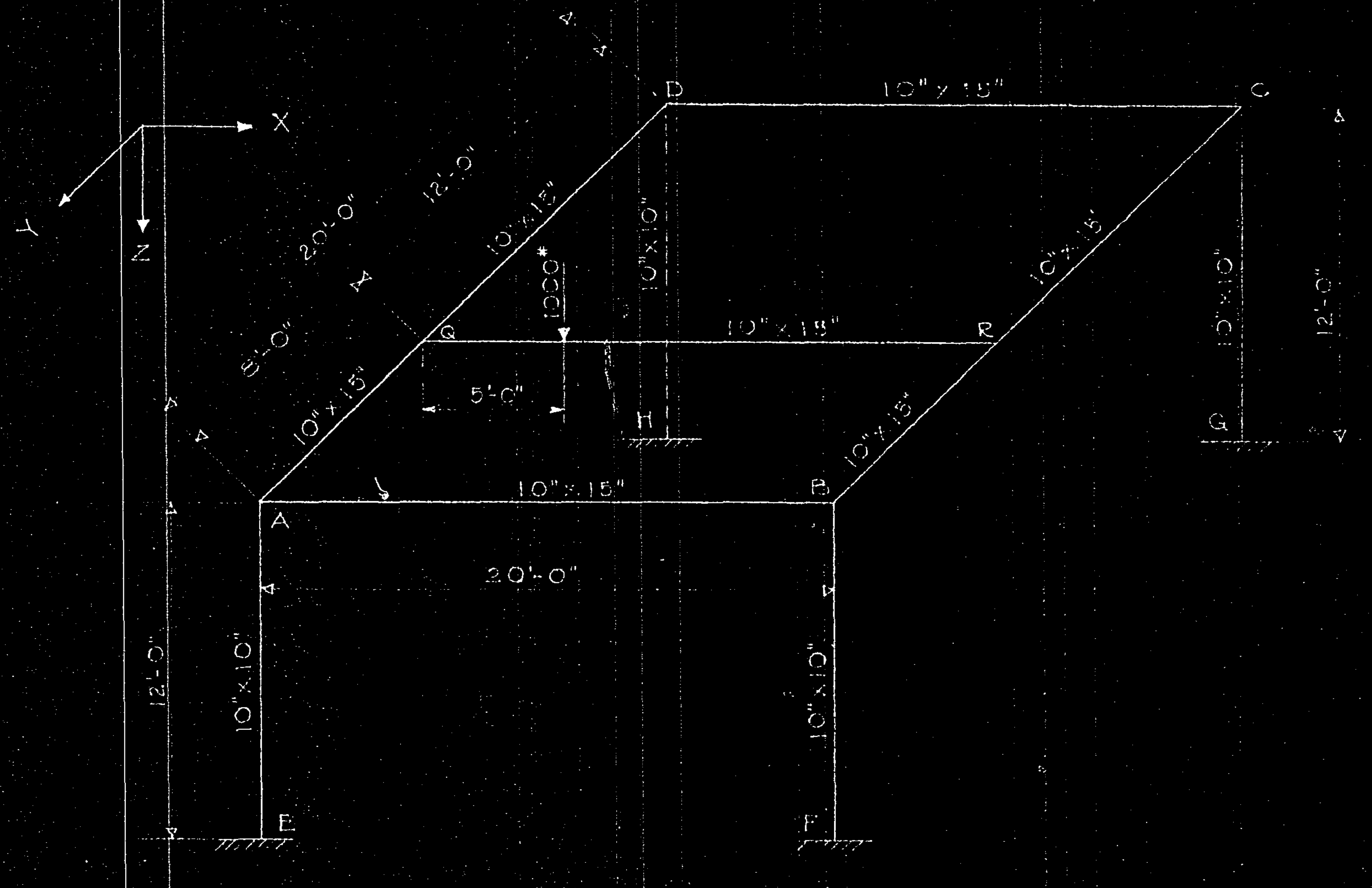


FIG. 13 FRAME B, DIMENSIONS

$$I_x = I_Y = \frac{10 \times \overline{10}^3}{12} = 833.33 \quad \text{FOR COLUMNS}$$

$$I_x = I_Y = \frac{10 \times \overline{15}^3}{12} = 2812.5 \quad \text{FOR BEAMS}$$

$$I_Z = \frac{15 \times \overline{10}^3}{12} = 1250 \quad \text{FOR BEAMS}$$

FOR COLUMNS

$$K = \frac{4EI}{L} = \frac{4E \times 833.33}{12 \times 12} = 23.15\,E \quad \text{(BENDING)}$$

$$K^* = \frac{k_1 G \times \overline{10}^4}{L} = \frac{0.1406 \times 0.4167E \times \overline{10}^4}{12 \times 12} = 4.07E \quad \text{(TORSION)}$$

FOR BEAMS 20'-0"

$$K = \frac{4EI}{L} = \frac{4E \times 1250}{20 \times 12} = 20.83E \quad \text{(BENDING)}$$

$$K = \frac{4EI}{L} = \frac{4E \times 2812.5}{20 \times 12} = 46.88E \quad \text{(BENDING)}$$

$$K^* = \frac{k_1 G (2a)^3 (2b)}{L} = \frac{0.196 \times 0.4167E \times (2 \times 5)^3 (2 \times 7.5)}{12 \times 12}$$

$$= 5.10E \quad \text{(TORSION)}$$

FOR BEAMS 12'-0"

$$K = \frac{4EI}{L} = \frac{4E \times 1250}{12 \times 12} = 34.72\,E \quad \text{(BENDING)}$$

$$K = \frac{4EI}{L} = \frac{4E \times 2812.5}{12 \times 12} = 78.13\,E \quad \text{(BENDING)}$$

$$K^* = \frac{k_1 G (2a)^3 (2b)}{L} = \frac{0.196 \times 0.4167E \times (2 \times 5)^3 (2 \times 7.5)}{12 \times 12}$$

$$= 8.51E \quad \text{(TORSION)}$$

FOR BEAMS 8'-0"

$$K = \frac{4EI}{L} = \frac{4E \times 1250}{8 \times 12} = 52.08E \quad \text{(BENDING)}$$

$$K = \frac{4EI}{L} = \frac{4E \times 2812.5}{8 \times 12} = 117.19E \quad \text{(BENDING)}$$

$$K^* = \frac{k_1 G (2a)^3 (2b)}{L} = \frac{0.196 \times 0.4167E \times (2 \times 5)^3 (2 \times 7.5)}{8 \times 12}$$

$$= 12.76\,E \quad \text{(TORSION)}$$

FIG. 14. FRAME B, CONSTANTS.

MOMENT DISTRIBUTION FACTORS

JOINT A

		AQ	AE	AB	
FOR M_x	K =	117.19 +	23.15 +	5.10 =	145.44
	D.F. =	0.806	0.158	0.035	
FOR M_y	K =	12.76 +	23.15 +	46.88 =	82.79
	D.F. =	0.154	0.280	0.566	
FOR M_z	K =	52.08 +	4.07 +	20.83 =	76.98
	D.F. =	0.676	0.053	0.271	

JOINT C

		CR	CG	CD	
FOR M_x	K =	78.13 +	23.15 +	5.10 =	106.38
	D.F. =	0.734	0.218	0.048	
FOR M_y	K =	8.51 +	23.15 +	46.88 =	78.54
	D.F. =	0.108	0.295	0.597	
FOR M_z	K =	34.72 +	4.07 +	20.83 =	59.62
	D.F. =	0.582	0.068	0.350	

JOINT Q

		QA	QD	QR	
FOR M_x	K =	117.19 +	78.13 +	5.10 =	200.42
	D.F. =	0.585	0.390	0.025	
FOR M_y	K =	12.76 +	8.51 +	46.88 =	68.15
	D.F. =	0.187	0.125	0.688	
FOR M_z	K =	52.08 +	34.72 +	20.83 =	107.63
	D.F. =	0.484	0.323	0.193	

BY SYMMETRY THE MOMENT DISTRIBUTION FACTORS FOR JOINTS B, D, & R ARE SIMILAR TO THOSE AT A, C, & Q RESPECTIVELY.

FIG. 14 - CONTINUED

SHEAR DISTRIBUTION FACTOR

$$\frac{1}{L^3} = \frac{1250}{12^3 \times 20^3} = 0.000090422$$

$$\frac{1}{L^3} = \frac{2812.5}{12^3 \times 20^3} = 0.000203451$$

$$\frac{1}{L^3} = \frac{1250}{12^3 \times 12^3} = 0.000418622$$

$$\frac{1}{L^3} = \frac{2812.5}{12^3 \times 12^3} = 0.000941901$$

$$\frac{1}{L^3} = \frac{833.33}{12^3 \times 12^3} = 0.000279081$$

$$\frac{1}{L^3} = \frac{1250}{8^3 \times 12^3} = 0.001412851$$

$$\frac{1}{L^3} = \frac{2812.5}{8^3 \times 12^3} = 0.003178914$$

SHEAR$_{AY}$

MEMBER	AB	QR	DC	AE	DH
S.D.F.	0.109017	0.109017	0.109017	0.336474	0.336474

SHEAR$_{CX}$

MEMBER	CR	DQ	CG	DH	
S.D.F.	0.300000	0.300000	0.200000	0.200000	

SHEAR$_{RX}$

MEMBER	RB	QA	RC	QD	
S.D.F.	0.385714	0.385714	0.114286	0.114286	

SHEAR$_{BX}$

MEMBER	BR	AQ	BF	AE	
S.D.F	0.417526	0.417526	0.082474	0.082474	

SHEAR$_{QZ}$

MEMBER	QA	QD	QR		
S.D.F.	0.735134	0.217817	0.047049		

BY SYMMETRY THE (SHEAR DISTRIBUTION FACTOR)$_{BY}$ AND (SHEAR DISTRIBUTION FACTOR)$_{RZ}$ ARE SIMILAR TO (SHEAR DISTRIBUTION FACTOR)$_{AY}$ AND (SHEAR DISTRIBUTION FACTOR)$_{QZ}$ RESPECTIVELY

FIG. 14:- CONTINUED

In this problem the shear-moment distribution method as applied to Frame A is not very convenient to use. This is true because the lengths of the members are not the same and because more members are involved; these conditions make the use of the shear-moment method cumbersome. However, a slight change in procedure will result in a more powerful tool for making sidesway corrections not only for continuous space frames but also for irregular continuous planar frames. The method to be described in detail will be designated as the "Shear Distribution Method" applied to the continuous space frame.

All the necessary constants required for the solution of Frame B are found in Figure 14. The moment distribution factors, are of course found in the usual manner. The shear distribution factors are found by dividing the shear stiffness factor, I/L^3, of one member by the sum of the shear stiffness factors of all the members involved, as described in the part II-C. For example, using the data from Figure 14, the shear stiffness factors at joint A of members AB, QR, DC are 0.000090422 and those of members AE and DH are 0.000279081. The sum of the shear stiffness factors is then 0.000829428. The shear distribution factors of members AB, QR, DC are, therefore, 0.109017 and those of members AE and DH are 0.336474. These shear distribution factors are for the distribution of the shear at A along the Y-axis.

With all the constants of Frame B determined, the method of shear distribution will now be discussed by the aid of Table 10. A 100 pound load is applied at A in the negative direction of the Y-axis. As the author said before, this 100 pound load sets up an equal and opposite unbalanced shear at A (see II-C-1). This unbalanced shear is distributed as follows: -33.647 to each of members AE and DH; -10.902 to each of members AB, QR, DC, remembering that joints A, Q and D are Rigid-Free joints, B, R and C are Rigid-Fixed. All these shears are carried over to the opposite ends with their signs changed. The shears, +33.647, carried to the fixed end of each column need not be considered any further. The shears, +10.902, in members BA, RQ, CD totaling +32.706 constitute the unbalanced shear at B along the Y-axis. This shear should be distributed as follows: -11.005 to each of members BF and CG; and -3.566 to each of members BA, RQ, CD, remembering that the joints B, R and C are Rigid-Free joints, and A, Q and D are Rigid-Fixed. These shears are carried over and redistributed as described before. The process is continued until the unbalanced shears are negligible. Observe that in these operations the shear-distribution factors and carry-over factors are all negative. The algebraic sum of the shears at the end of each member gives the shear induced by the applied load when the joints A, B, C, D, Q, R are in a Rigid-Free condition.

With these induced shears the induced moments can be determined at both ends of each member in the usual manner. These moments with proper signs are shown in Table 10. One obtains the shear moment factors by dividing the induced moments by 100, since the applied load is 100. The term $S.M.F._{AY}^{(-)}$ is interpreted to mean that the load was applied in the negative direction at joint A along the Y-axis.

The shear moment factors for the other joints are obtained in a similar manner. Their values may be found in Table 12.

All the shear moment factors having been determined, the final solution of Frame B is to be explained. The reader should refer to Figure 13 and Table 11. A 1000 pound load is now applied at the left quarter point of member QR. The fixed end moments are -2812.5 ft.-lb. at Q and +937.5 ft.-lb. at R. With all joints Hinged-Fixed, these end moments are distributed and carried over throughout the frame in the usual manner. At this stage, the shears at the ends of each member are computed as follows:

At Q	At R
-2812.5	+ 937.5
+1935.0	- 645.0
- 322.5	+ 967.5
-1200.0 ft.-lb.	+1260.0 ft.-lb.

The algebraic sum of the shears resulting from these moments and the simple beam reactions gives upward unbalanced shears of 747.00 pounds at Q and 253.00 pounds at R.

These shears are corrected by the use of the appropriate shear moment factors. In Table 12, these corrections are made by multiplying the unbalanced shears at Q and at R by their corresponding shear moment factors. The summations of these corrections are transferred to Table 11.

The carry-over moments and the shear moments form the new unbalanced moments at each joint. These are distributed at each joint and carried over to the far end of each member in the usual manner. For the convenience of the reader a set of shears is computed for another cycle for values taken from Table 11 and shown on the next page.

M_x	M_x	M_x	M_x	M_x	M_x	M_y	M_y	M_y
AE	BF	QA	QD	RB	RC	CG	BF	QR
DH	CG					DH	AE	
+ 355.6	+ 135.0	+ 726.9	+ 484.6	+ 276.0	+ 184.0	- 34.6	- 49.1	+ 69.2
+ 177.8	+ 67.5	+ 901.4	- 364.8	+ 342.3	- 138.5	- 17.3	- 24.6	- 409.2
- 216.7	- 82.3	+1802.7	- 729.6	+ 684.5	- 277.0	+103.7	+147.2	- 818.3
- 108.4	- 41.2	+ 363.5	+ 242.3	+ 138.0	+ 92.0	+ 51.9	+ 73.6	+ 34.6
+ 208.3	+ 79.0	+3794.5	- 367.5	+1440.8	- 139.5	+103.7	+147.1	-1123.7
s=17.358	s=6.583	s=474.313	s=30.625	s=180.100	s=11.625	s=8.642	s=12.258	s=56.185
$SM_{AY}^{-17.358}$*	$SM_{BY}^{-6.583}$	$SM_{QZ}^{+474.313}$	$SM_{QZ}^{+30.625}$	$SM_{RZ}^{+180.100}$	$SM_{RZ}^{+11.625}$	$SM_{CX}^{+8.642}$	$SM_{BX}^{+12.258}$	$SM_{QZ}^{+56.185}$ / $SM_{RZ}^{-56.185}$

*The exponents of the S.M. values have the opposite sign to the unbalanced shears at the joints.

In order to secure the new shear moments in Table 12, first add algebraically the exponents of the SM values in the last row of the preceding tabulation, giving:

$$SM_{AY}^{-17.358} \qquad SM_{BY}^{-6.583} \qquad SM_{CX}^{+8.642}$$

$$SM_{BX}^{+12.258} \qquad SM_{QZ}^{+561.123} \qquad SM_{RZ}^{+135.540}$$

From this point on the foregoing procedure may be followed until both shears and moments become negligible. The final solution for Frame B is obtained by adding, algebraically, the moments at the end of each member as given in Table 11.

The effect of shear along the X-axis is given in Figure 15; along the Y-axis in Figure 16; and along the Z-axis in Figure 17. The composite superimposed reactions at the supports and moments as well as torques at the ends of each member are given in Figure 18.

The summation of moments and shears with reference to the X-axis, Y-axis, and Z-axis indicates that Frame B is in static equilibrium for the external load and reactions.

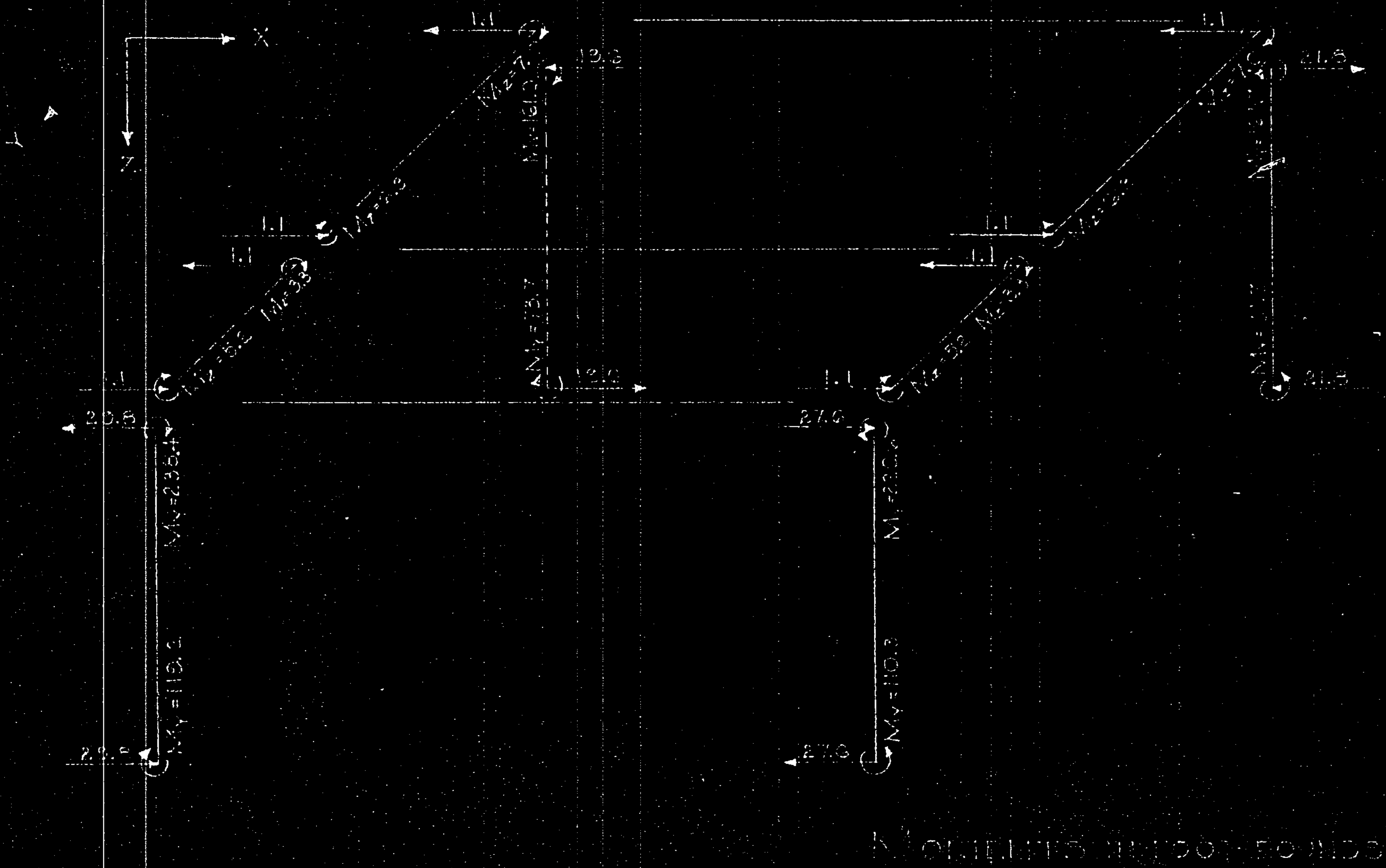
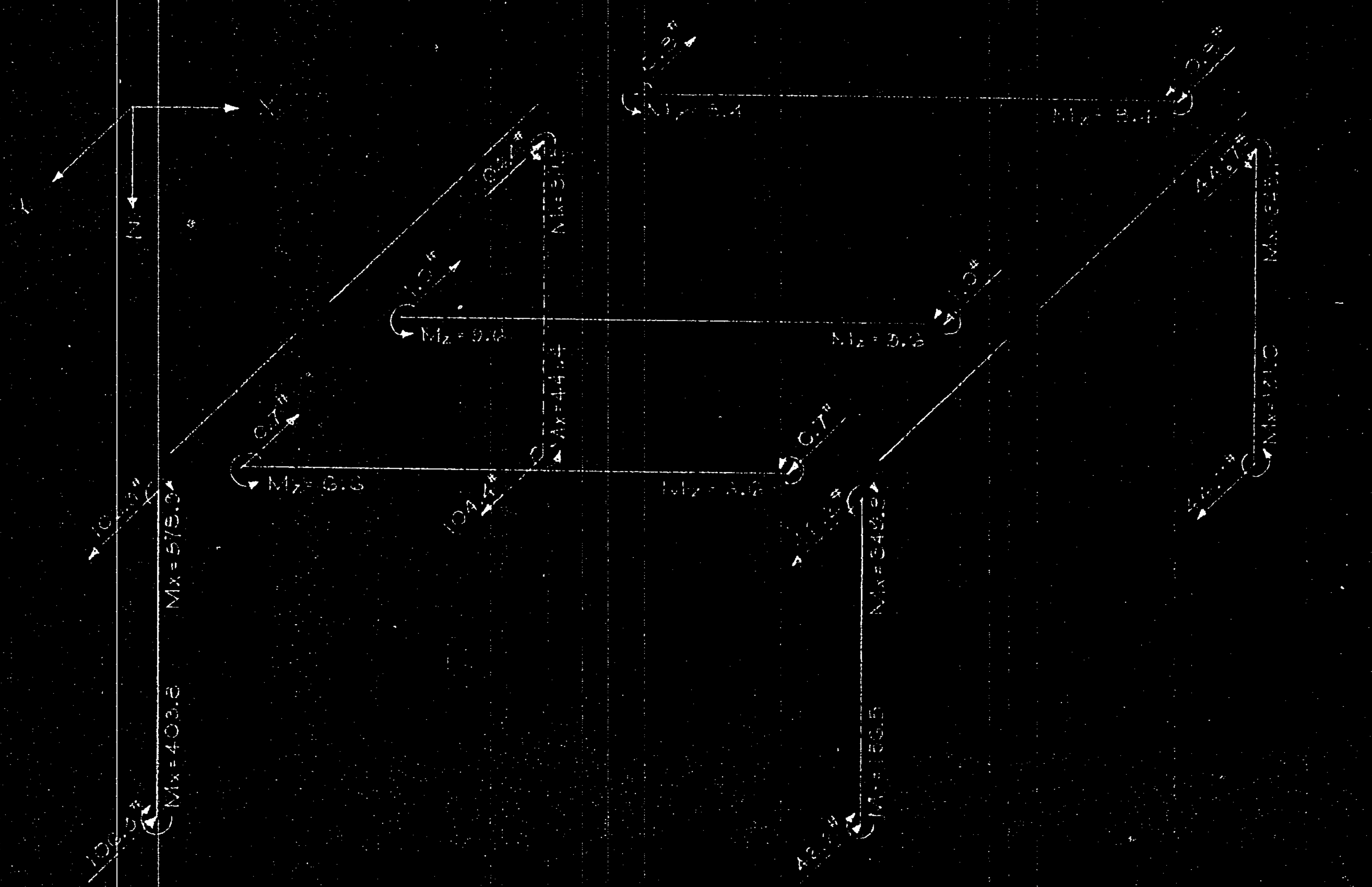
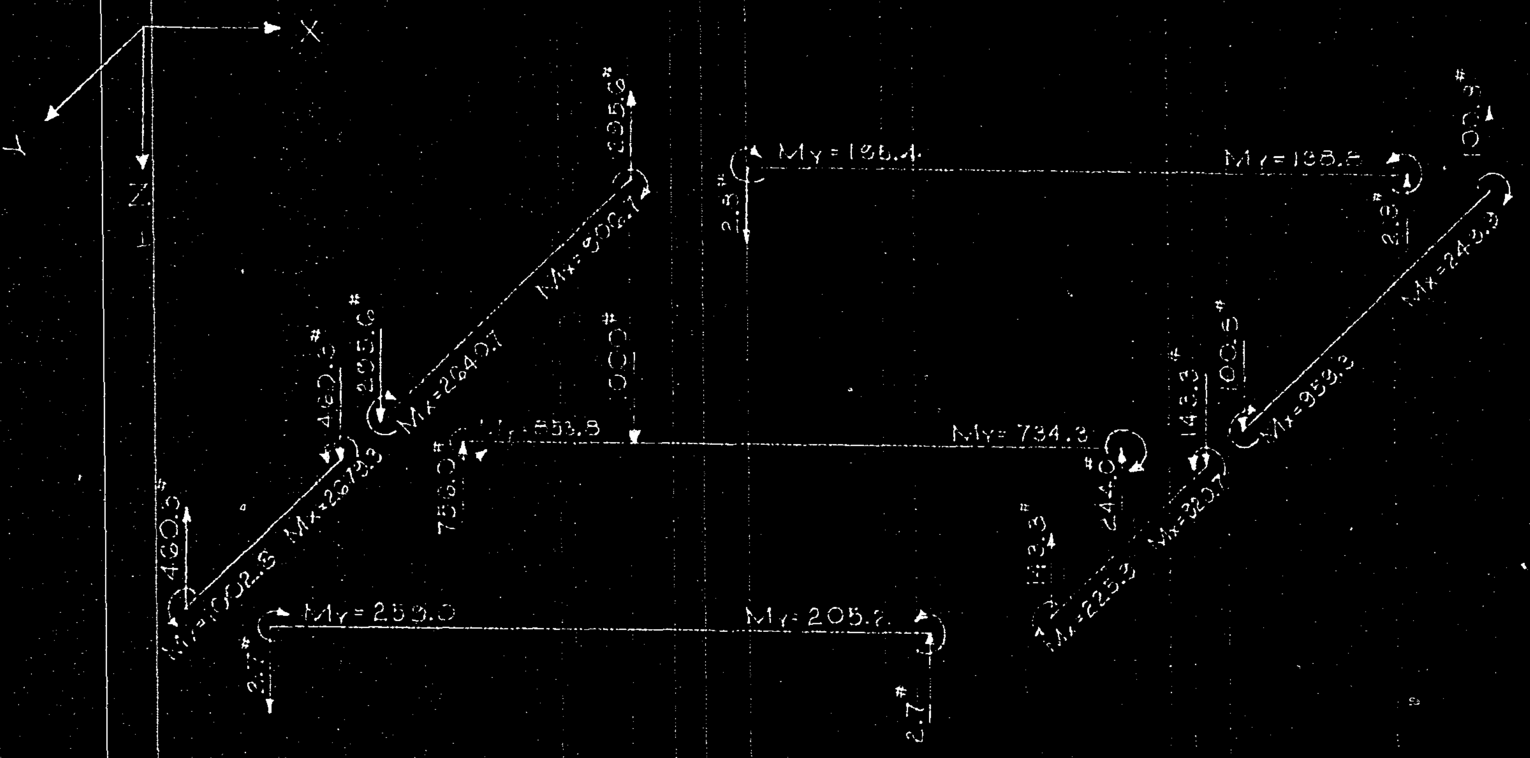


FIG. 15 SHEAR ALONG X-AXIS, FRAME B



MOMENTS IN FOOT POUNDS

FIG. 16 SHEAR ALONG Y-AXIS, FRAME B



MOMENTS IN FOOT-POUNDS

FIG. 17 SHEAR ALONG Z-AXIS, FRAME B

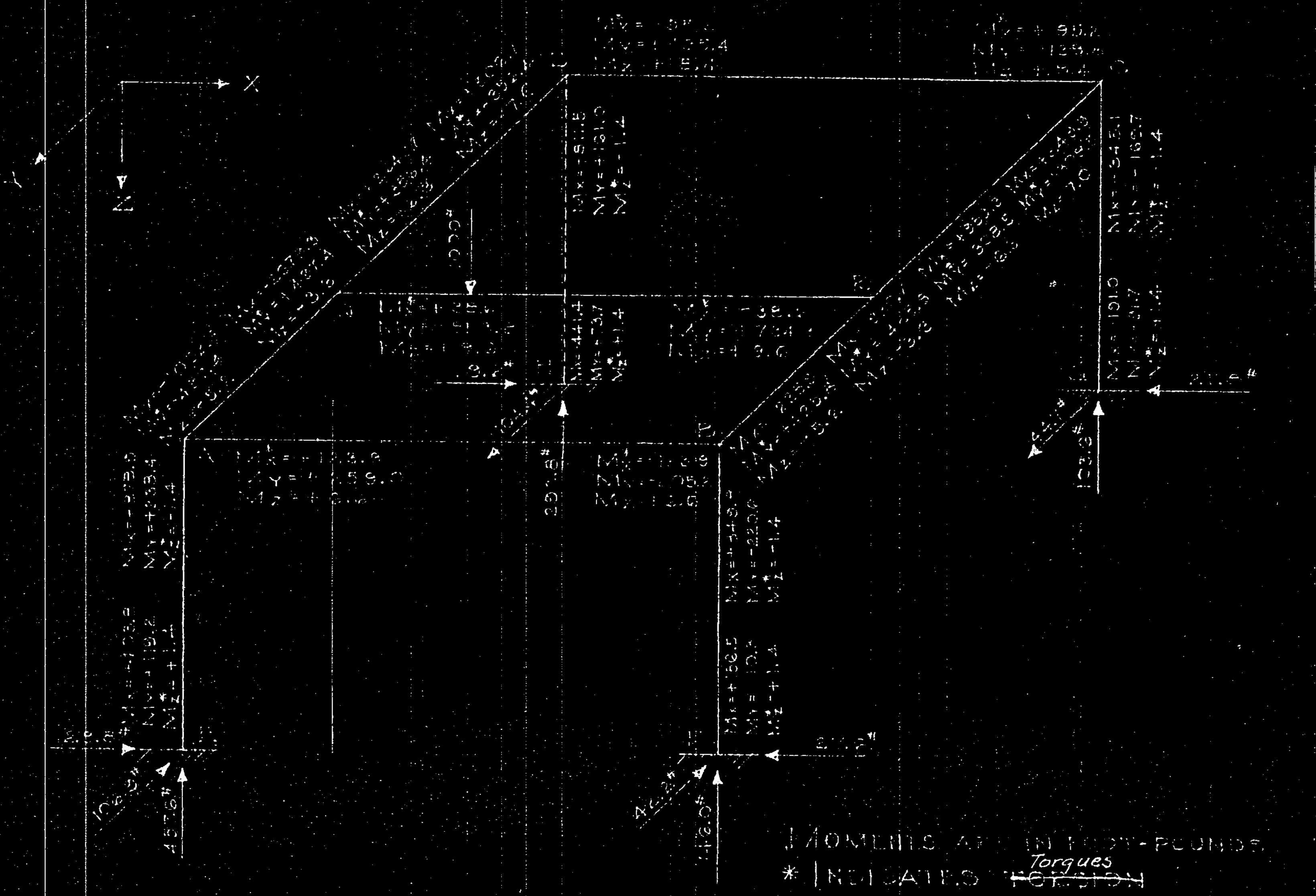


FIG. 18. SOLUTION FOR FRAME B

III. THREE-DIMENSIONAL SLOPE-DEFLECTION METHOD

The three-dimensional application of the slope-deflection method is presented with two purposes in mind: First, to show that the historical method of slope-deflections is applicable to the solution of a continuous space frame. Second, to introduce a check of the method developed by the author for those who have greater confidence in the established slope-deflection equations.

A. Nomenclature and Conventional Signs

Since three axes of reference should be considered, the nomenclature and conventional signs necessary for this type of work should be defined. The reader's attention is called to Table 13 for the sign convention in conjunction with the nomenclature listed as follows:

1. θ_{AX}, θ_{AY}, θ_{AZ}; θ_{BX}, θ_{BY}, θ_{BZ}; θ_{CX}, θ_{CY}, θ_{CZ};.... are angles of rotation at points A, B, C,.... in planes perpendicular to the axes of X, Y and Z, respectively.
2. The counterclockwise rotations, when one looks toward the positive directions of the axes X, Y and Z, are considered to be positive, while clockwise rotations are considered to be negative.

3. D_{AX}, D_{AY}, D_{AZ}; D_{BX}, D_{BY}, D_{BZ}; D_{CX}, D_{CY}, D_{CZ};.... are deflections at points A, B, C,.... along the axes X, Y and Z, respectively.

4. Deflections in the positive directions of the axes X, Y and Z are considered to be positive, and those in the negative directions are negative.

5. M_{ABX}, M_{ABY}, M_{ABZ}; M_{BAX}, M_{BAY}, M_{BAZ}; M_{BCX}, M_{BCY}, M_{BCZ}; M_{CBX}, M_{CBY}, M_{CBZ};.... are moments on members AB, BA, BC, CB,.... at ends A, B, C,.... with respect to the axes X, Y and Z, respectively.

6. M_{ABX}, M_{ABY}, M_{ABZ}; M_{BAX}, M_{BAY}, M_{BAZ}; are considered to be positive when they tend to rotate the joints clockwise as one looks toward the positive direction of the axes X, Y and Z, respectively.

7. S_{AX}, S_{AY}, S_{AZ}; S_{BX}, S_{BY}, S_{BZ}; S_{CX}, S_{CY}, S_{CZ};.... are shears at points A, B, C,.... along the axes X, Y and Z, respectively.

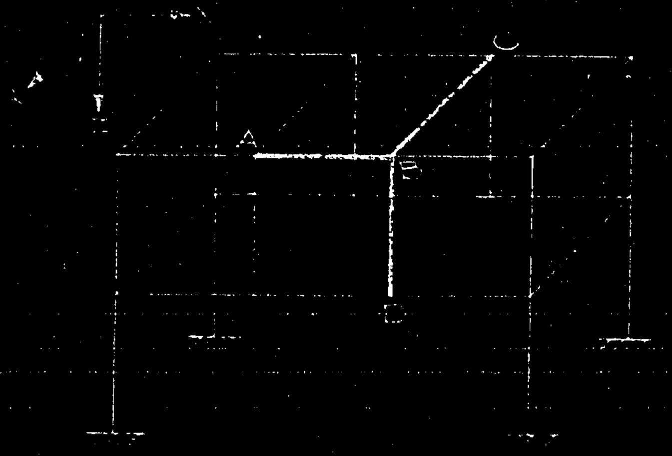
8. S_{AX}, S_{AY}, S_{AZ}; S_{BX}, S_{BY}, S_{BZ};.... are positive when they are in the positive directions of the axes X, Y and Z, respectively.

9. M_{AX}, M_{AY}, M_{AZ}; M_{BX}, M_{BY}, M_{BZ}; M_{CX}, M_{CY}, M_{CZ};.... are moments at points A, B, C,.... with respect to the axes X, Y and Z, respectively.

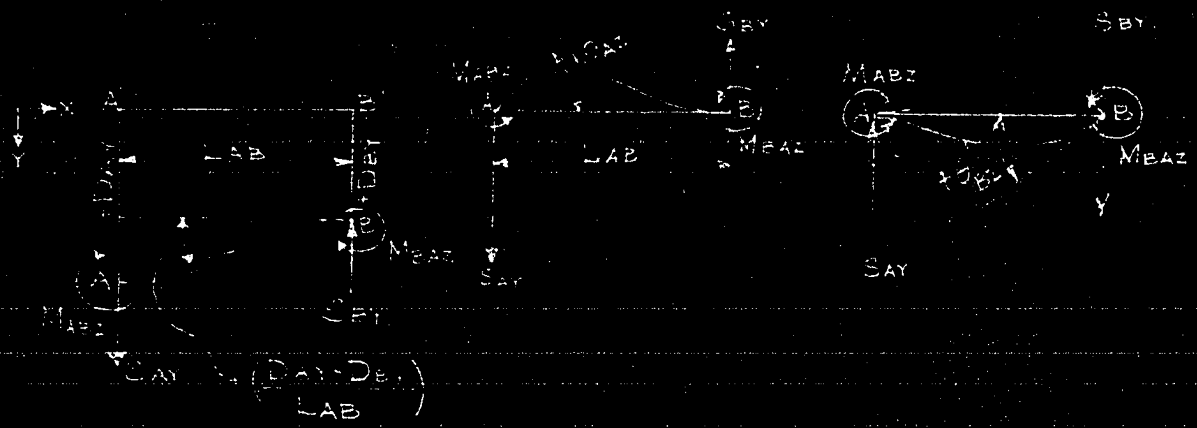
B. Typical Moment and Torque Expressions

The reader's attention is again called to Table 13. Since the relationships of the end moments of a member with respect to the rotation of the ends and the deflection of one end relative to the other end are given in standard textbooks, no attempt is made to prove them here.

TABLE 13 TYPICAL MOMENT AND TORQUE EXPRESSIONS



NOTE: FOR A LOADED MEMBER THE FIXED END MOMENT IS ADDED TO EACH OF THE FOLLOWING TYPICAL EQUATIONS EXCEPT THOSE FOR TORQUE.



$$M_{ABZ} = -\frac{6EI_z}{L_{AB}}\left(\frac{D_{AY}-D_{BY}}{L_{AB}}\right) \qquad M_{ABZ} = +\left(\frac{4EI_z}{L_{AB}}\right)\theta_{AZ} \qquad M_{ABZ} = +\left(\frac{2EI_z}{L_{AB}}\right)\theta_{BZ}$$

$$M_{BAZ} = -\frac{6EI_z}{L_{AB}}\left(\frac{D_{AY}-D_{BY}}{L_{AB}}\right) \qquad M_{BAZ} = +\left(\frac{2EI_z}{L_{AB}}\right)\theta_{AZ} \qquad M_{BAZ} = +\left(\frac{4EI_z}{L_{AB}}\right)\theta_{BZ}$$

SIGN CONVENTION OF SLOPE-DEFLECTION EQUATIONS

A

TABLE 13 CONTINUED

MEMBER AB

$$M^*_{ABX} = \frac{K_1 G (2a)^3 (2b)}{L_{AB}} \left(\theta_{AX} - \theta_{BX}\right) \tag{1}$$

$$M_{ABY} = 2E\left(\frac{I_Y}{L_{AB}}\right)\left[2\theta_{AY} + \theta_{BY} - 3\left(\frac{D_{BZ} - D_{AZ}}{L_{AB}}\right)\right] \tag{2}$$

$$M_{ABZ} = 2E\left(\frac{I_Z}{L_{AB}}\right)\left[2\theta_{AZ} + \theta_{BZ} - 3\left(\frac{D_{AY} - D_{BY}}{L_{AB}}\right)\right] \tag{3}$$

$$M^*_{BAX} = \frac{K_1 G (2a)^3 (2b)}{L_{AB}} \left(\theta_{BX} - \theta_{AX}\right) \tag{4}$$

$$M_{BAY} = 2E\left(\frac{I_Y}{L_{AB}}\right)\left[2\theta_{BY} + \theta_{AY} - 3\left(\frac{D_{BZ} - D_{AZ}}{L_{AB}}\right)\right] \tag{5}$$

$$M_{BAZ} = 2E\left(\frac{I_Z}{L_{AB}}\right)\left[2\theta_{BZ} + \theta_{AZ} - 3\left(\frac{D_{AY} - D_{BY}}{L_{AB}}\right)\right] \tag{6}$$

MEMBER BC

$$M_{BCX} = 2E\left(\frac{I_X}{L_{BC}}\right)\left[2\theta_{BX} + \theta_{CX} - 3\left(\frac{D_{CZ} - D_{BZ}}{L_{BC}}\right)\right] \tag{7}$$

$$M^*_{BCY} = \frac{K_1 G (2a)^3 (2b)}{L_{BC}} \left(\theta_{BY} - \theta_{CY}\right) \tag{8}$$

$$M_{BCZ} = 2E\left(\frac{I_Z}{L_{BC}}\right)\left[2\theta_{BZ} + \theta_{CZ} - 3\left(\frac{D_{BX} - D_{CX}}{L_{BC}}\right)\right] \tag{9}$$

$$M_{CBX} = 2E\left(\frac{I_X}{L_{BC}}\right)\left[2\theta_{CX} + \theta_{BX} - 3\left(\frac{D_{CZ} - D_{BZ}}{L_{BC}}\right)\right] \tag{10}$$

$$M^*_{CBY} = \frac{K_1 G (2a)^3 (2b)}{L_{BC}} \left(\theta_{CY} - \theta_{BY}\right) \tag{11}$$

$$M_{CBZ} = 2E\left(\frac{I_Z}{L_{BC}}\right)\left[2\theta_{CZ} + \theta_{BZ} - 3\left(\frac{D_{BX} - D_{CX}}{L_{BC}}\right)\right] \tag{12}$$

MEMBER BD

$$M_{BDX} = 2E\left(\frac{I_X}{L_{BD}}\right)\left[2\theta_{BX} + \theta_{DX} - 3\left(\frac{D_{DY} - D_{BY}}{L_{BD}}\right)\right] \tag{13}$$

$$M_{BDY} = 2E\left(\frac{I_Y}{L_{BD}}\right)\left[2\theta_{BY} + \theta_{DY} - 3\left(\frac{D_{BX} - D_{DX}}{L_{BD}}\right)\right] \tag{14}$$

$$M^*_{BDZ} = \frac{K_1 G (2a)^3 (2b)}{L_{BD}} \left(\theta_{BZ} - \theta_{DZ}\right) \tag{15}$$

$$M_{DBX} = 2E\left(\frac{I_X}{L_{BD}}\right)\left[2\theta_{DX} + \theta_{BX} - 3\left(\frac{D_{DY} - D_{BY}}{L_{BD}}\right)\right] \tag{16}$$

$$M_{DBY} = 2E\left(\frac{I_Y}{L_{BD}}\right)\left[2\theta_{DY} + \theta_{BY} - 3\left(\frac{D_{BX} - D_{DX}}{L_{BD}}\right)\right] \tag{17}$$

$$M^*_{DBZ} = \frac{K_1 G (2a)^3 (2b)}{L_{BD}} \left(\theta_{DZ} - \theta_{BZ}\right) \tag{18}$$

However, one should notice that the moments derived from deflections are given in terms of the difference of the deflections at both ends of the members. The rule for the formulation of this term is to imagine that the positive axis of reference for the moments under consideration is perpendicular and pointed to the back of the plane in which the deflections lie. When the positive direction of deflections along the shear axis is downward, then one should subtract the deflection at right from that at the left.

For the convenience of the reader typical expressions are written for members AB, BC and BD as given in Table 13.

The expression for torques in terms of the angle of twist is obtained from "Theory of Elasticity" by S. Timoshenko, p. 248, McGraw-Hill Book Company, Inc., New York, 1934.

It is convenient to assume that all unknowns are positive. The negative values will be singled out automatically by the minus signs in the final solution.

C. Application to Typical Example, Frame B

In order to show the consistency of the results obtained from the author's three-dimensional application of the moment-distribution method, Frame B has been selected as a general, typical example to be solved by the application of the slope-deflection equations. So far as known, this presentation is also an original solution by the author.

The major problem in the three-dimensional slope-deflection method is to set up the correct equations. The final solution involves skill in solving simultaneous equations. The author, therefore, has concentrated upon the formulation of the correct simultaneous equations. The adequacy of these equations can be demonstrated by substituting the known values obtained by the foregoing three-dimensional moment-distribution method.

1. Moment-torque expressions

These expressions are set up in terms of the nomenclature and conventional signs presented in part III-A. They are given completely in Table 14, for each member of the Frame B. For convenience in setting up the equations all rotations and slopes are considered to be positive.

2. Equilibrium conditions and equations

The equilibrium conditions of the moments which must be maintained at each joint provide eighteen equations. Since there are twenty-five unknowns, seven more equations have to be set up by the consideration of the equilibrium of the shears at each joint. Equation 19, in Table 15, will serve to illustrate this part of the procedure. Shears are considered as positive when they point along the positive direction of the axes of reference. By considering the moments on all members to be positive the shear along the Y-axis of member AE at point A is positive; the same sign applies to member DH at point A. When there is no external load at a joint the sum of the shears at the joint must equal zero; this fact is shown in equations 19 to 25, inclusive.

TABLE-14 MOMENT AND TORQUE EXPRESSIONS FOR FRAME B

ASSUMING ALL ROTATIONS AND ALL DEFLECTIONS ARE POSITIVE, THE MOMENTS AND TORQUES CAN BE EXPRESSED AS FOLLOWS:

JOINT A

$$M_{AQX} = 2E\left(\frac{I_X}{L_{AQ}}\right)\left[2\theta_{AX} + \theta_{QX} - 3\left(\frac{D_{QZ}}{L_{AQ}}\right)\right] \qquad (1)$$

$$M^{*}_{AQY} = \frac{k_1 G (2a)^3 (2b)}{L_{AQ}}\left(\theta_{AY} - \theta_{QY}\right) \qquad (2)$$

$$M_{AQZ} = 2E\left(\frac{I_Z}{L_{AQ}}\right)\left[2\theta_{AZ} + \theta_{QZ} - 3\left(\frac{D_{QX} - D_{AX}}{L_{AQ}}\right)\right] \qquad (3)$$

$$M_{AEX} = 2E\left(\frac{I_X}{L_{AE}}\right)\left[2\theta_{AX} + 3\left(\frac{D_{AY}}{L_{AE}}\right)\right] \qquad (4)$$

$$M_{AEY} = 2E\left(\frac{I_Y}{L_{AE}}\right)\left[2\theta_{AY} - 3\left(\frac{D_{AX}}{L_{AE}}\right)\right] \qquad (5)$$

$$M^{*}_{AEZ} = \frac{k_1 G (2a)^3 (2b)}{L_{AE}}\left(\theta_{AZ}\right) \qquad (6)$$

$$M^{*}_{ABX} = \frac{k_1 G (2a)^3 (2b)}{L_{AB}}\left(\theta_{AX} - \theta_{BX}\right) \qquad (7)$$

$$M_{ABY} = 2E\left(\frac{I_Y}{L_{AB}}\right)\left[2\theta_{AY} + \theta_{BY}\right] \qquad (8)$$

$$M_{ABZ} = 2E\left(\frac{I_Z}{L_{AB}}\right)\left[2\theta_{AZ} + \theta_{BZ} - 3\left(\frac{D_{AY} - D_{BY}}{L_{AB}}\right)\right] \qquad (9)$$

TABLE-14 CONTINUED

JOINT B

$$M_{BRX} = 2E\left(\frac{I_x}{L_{BR}}\right)\left[2\Theta_{BX} + \Theta_{RX} - 3\left(\frac{D_{RZ}}{L_{BR}}\right)\right] \quad (10)$$

$$M^{*}_{BRY} = \frac{K_1 G\,(2a)^3(2b)}{L_{BR}}\left(\Theta_{BY} - \Theta_{RY}\right) \quad (11)$$

$$M_{BRZ} = 2E\left(\frac{I_z}{L_{BR}}\right)\left[2\Theta_{BZ} + \Theta_{RZ} - 3\left(\frac{D_{BX} - D_{RX}}{L_{BR}}\right)\right] \quad (12)$$

$$M_{BFX} = 2E\left(\frac{I_x}{L_{BF}}\right)\left[2\Theta_{BX} + 3\left(\frac{D_{BY}}{L_{BF}}\right)\right] \quad (13)$$

$$M_{BFY} = 2E\left(\frac{I_y}{L_{BF}}\right)\left[2\Theta_{BY} - 3\left(\frac{D_{BX}}{L_{BF}}\right)\right] \quad (14)$$

$$M^{*}_{BFZ} = \frac{K_1 G\,(2a)^3(2b)}{L_{BF}}\left(\Theta_{BZ}\right) \quad (15)$$

$$M^{*}_{BAX} = \frac{K_1 G\,(2a)^3(2b)}{L_{AB}}\left(\Theta_{BX} - \Theta_{AX}\right) \quad (16)$$

$$M_{BAY} = 2E\left(\frac{I_Y}{L_{AB}}\right)\left[2\Theta_{BY} + \Theta_{AY}\right] \quad (17)$$

$$M_{BAZ} = 2E\left(\frac{I_z}{L_{AB}}\right)\left[2\Theta_{BZ} + \Theta_{AZ} - 3\left(\frac{D_{AY} - D_{BY}}{L_{AB}}\right)\right] \quad (18)$$

TABLE 14 CONTINUED

JOINT C

$$M_{CRX} = 2E\left(\frac{I_X}{L_{CR}}\right)\left[2\theta_{CX} + \theta_{RX} + 3\left(\frac{D_{RZ}}{L_{CR}}\right)\right] \quad (19)$$

$$M^*_{CRY} = \frac{k_1 G (2a)^3 (2b)}{L_{CR}}\left(\theta_{CY} - \theta_{RY}\right) \quad (20)$$

$$M_{CRZ} = 2E\left(\frac{I_Z}{L_{CR}}\right)\left[2\theta_{CZ} + \theta_{RZ} - 3\left(\frac{D_{RX} - D_{CX}}{L_{CR}}\right)\right] \quad (21)$$

$$M_{CGX} = 2E\left(\frac{I_X}{L_{CG}}\right)\left[2\theta_{CX} + 3\left(\frac{D_{EX}}{L_{CG}}\right)\right] \quad (22)$$

$$M_{CGY} = 2E\left(\frac{I_Y}{L_{CG}}\right)\left[2\theta_{CY} - 3\left(\frac{D_{CX}}{L_{CG}}\right)\right] \quad (23)$$

$$M^*_{CGZ} = \frac{k_1 G (2a)^3 (2b)}{L_{CG}}\left(\theta_{CZ}\right) \quad (24)$$

$$M^*_{CDX} = \frac{k_1 G (2a)^3 (2b)}{L_{CD}}\left(\theta_{CX} - \theta_{DX}\right) \quad (25)$$

$$M_{CDY} = 2E\left(\frac{I_Y}{L_{CD}}\right)\left[2\theta_{CY} + \theta_{DY}\right] \quad (26)$$

$$M_{CDZ} = 2E\left(\frac{I_Z}{L_{CD}}\right)\left[2\theta_{CZ} + \theta_{DZ} - 3\left(\frac{D_{AY} - D_{BY}}{L_{CD}}\right)\right] \quad (27)$$

C

TABLE-14 CONTINUED

JOINT D

$$M_{DQX} = 2E\left(\frac{I_X}{L_{DQ}}\right)\left[2\theta_{DX}+\theta_{QX}+3\left(\frac{D_{QZ}}{L_{DQ}}\right)\right] \quad (28)$$

$$M^{*}_{DQY} = \frac{k_1 G (2a)^3 (2b)}{L_{DQ}}\left(\theta_{DY}-\theta_{QY}\right) \quad (29)$$

$$M_{DQZ} = 2E\left(\frac{I_Z}{L_{DQ}}\right)\left[2\theta_{DZ}+\theta_{QZ}-3\left(\frac{D_{RX}-D_{CX}}{L_{DQ}}\right)\right] \quad (30)$$

$$M_{DHX} = 2E\left(\frac{I_X}{L_{DH}}\right)\left[2\theta_{DX}+3\left(\frac{D_{AY}}{L_{DH}}\right)\right] \quad (31)$$

$$M_{DHY} = 2E\left(\frac{I_Y}{L_{DH}}\right)\left[2\theta_{DY}-3\left(\frac{D_{CX}}{L_{DH}}\right)\right] \quad (32)$$

$$M^{*}_{DHZ} = \frac{k_1 G (2a)^3 (2b)}{L_{DH}}\left(\theta_{DZ}\right) \quad (33)$$

$$M^{*}_{DCX} = \frac{k_1 G (2a)^3 (2b)}{L_{CD}}\left(\theta_{DX}-\theta_{CX}\right) \quad (34)$$

$$M_{DCY} = 2E\left(\frac{I_Y}{L_{CD}}\right)\left[2\theta_{DY}+\theta_{CY}\right] \quad (35)$$

$$M_{DCZ} = 2E\left(\frac{I_Z}{L_{CD}}\right)\left[2\theta_{DZ}+\theta_{CZ}-3\left(\frac{D_{AY}-D_{BY}}{L_{CD}}\right)\right] \quad (36)$$

D

TABLE-14 CONTINUED

JOINT Q

$$M_{QAX} = 2E\left(\frac{I_x}{L_{AQ}}\right)\left[2\theta_{QX} + \theta_{AX} - 3\left(\frac{D_{QZ}}{L_{AQ}}\right)\right] \quad (37)$$

$$M^{s}_{QAY} = \frac{K_1 G (2a)^3 (2b)}{L_{AQ}}\left(\theta_{QY} - \theta_{AY}\right) \quad (38)$$

$$M_{QAZ} = 2E\left(\frac{I_z}{L_{AQ}}\right)\left[2\theta_{QZ} + \theta_{AZ} - 3\left(\frac{D_{QX} - D_{AX}}{L_{AQ}}\right)\right] \quad (39)$$

$$M_{QDX} = 2E\left(\frac{I_x}{L_{DQ}}\right)\left[2\theta_{QX} + \theta_{DX} + 3\left(\frac{D_{QZ}}{L_{DQ}}\right)\right] \quad (40)$$

$$M^{s}_{QDY} = \frac{K_1 G (2a)^3 (2b)}{L_{DQ}}\left(\theta_{QY} - \theta_{DY}\right) \quad (41)$$

$$M_{QDZ} = 2E\left(\frac{I_z}{L_{DQ}}\right)\left[2\theta_{QZ} + \theta_{DZ} - 3\left(\frac{D_{DX} - D_{QX}}{L_{DQ}}\right)\right] \quad (42)$$

$$M^{*}_{QRY} = \frac{K_1 G (2a)^3 (2b)}{L_{QR}}\left(\theta_{QY} - \theta_{RY}\right) \quad (43)$$

$$M_{QRY} = 2E\left(\frac{I_y}{L_{QR}}\right)\left[2\theta_{QY} + \theta_{RY} - 3\left(\frac{D_{RZ} - D_{QZ}}{L_{QR}}\right) - M^{s}_{QRY}\right] \quad (44)$$

$$M_{QRZ} = 2E\left(\frac{I_z}{L_{QR}}\right)\left[2\theta_{QZ} + \theta_{RZ} - 3\left(\frac{D_{RX} - D_{QX}}{L_{QR}}\right)\right] \quad (45)$$

E

TABLE-14 CONTINUED

JOINT R

$$M_{RBX} = 2E\left(\frac{I_x}{L_{BR}}\right)\left[2\,\Theta_{RX} + \Theta_{BX} - 3\left(\frac{D_{RZ}}{L_{BR}}\right)\right] \quad (46)$$

$$M^{*}_{RBY} = \frac{K_1 G (2a)^3 (2b)}{L_{BR}}\left(\Theta_{RY} - \Theta_{BY}\right) \quad (47)$$

$$M_{RBZ} = 2E\left(\frac{I_z}{L_{BR}}\right)\left[2\,\Theta_{RZ} + \Theta_{BZ} - 3\left(\frac{D_{BX} - D_{RX}}{L_{BR}}\right)\right] \quad (48)$$

$$M_{RCX} = 2E\left(\frac{I_x}{L_{CR}}\right)\left[2\,\Theta_{RX} + \Theta_{CX} + 3\left(-\frac{D_{RZ}}{L_{CR}}\right)\right] \quad (49)$$

$$M^{*}_{RCY} = \frac{K_1 G (2a)^3 (2b)}{L_{CR}}\left(\Theta_{RY} - \Theta_{CY}\right) \quad (50)$$

$$M_{RCZ} = 2E\left(\frac{I_z}{L_{CR}}\right)\left[2\,\Theta_{RZ} + \Theta_{CZ} - 3\left(\frac{D_{RX} - D_{CX}}{L_{CR}}\right)\right] \quad (51)$$

$$M^{*}_{RQX} = \frac{K_1 G (2a)^3 (2b)}{L_{QR}}\left(\Theta_{RX} - \Theta_{QX}\right) \quad (52)$$

$$M^{*}_{RQY} = 2E\left(\frac{I_y}{L_{QR}}\right)\left[2\,\Theta_{RY} + \Theta_{QY} - 3\left(\frac{D_{RZ} - D_{QZ}}{L_{QR}}\right)\right] + M^{F}_{RQY} \quad (53)$$

$$M_{RQZ} = 2E\left(\frac{I_z}{L_{QR}}\right)\left[2\,\Theta_{RZ} + \Theta_{QZ} - 3\left(\frac{D_{AY} - D_{BY}}{L_{QR}}\right)\right] \quad (54)$$

TABLE 14 - CONTINUED

JOINT E

$$M_{EAX} = 2E\left(\frac{I_X}{L_{AE}}\right)\left[\theta_{AX} + 3\left(\frac{D_{AY}}{L_{AE}}\right)\right] \qquad (55)$$

$$M_{EAY} = 2E\left(\frac{I_Y}{L_{AE}}\right)\left[\theta_{AY} - 3\left(\frac{D_{BX}}{L_{AE}}\right)\right] \qquad (56)$$

$$M^{*}_{EAZ} = \frac{K_1 G (2a)^3 (2b)}{L_{AE}}\left(-\theta_{AZ}\right) \qquad (57)$$

JOINT F

$$M_{FBX} = 2E\left(\frac{I_X}{L_{BF}}\right)\left[\theta_{BX} + 3\left(\frac{D_{BY}}{L_{BF}}\right)\right] \qquad (58)$$

$$M_{FBY} = 2E\left(\frac{I_Y}{L_{BF}}\right)\left[\theta_{BY} - 3\left(\frac{D_{BX}}{L_{BF}}\right)\right] \qquad (59)$$

$$M^{*}_{FBZ} = \frac{K_1 G (2a)^3 (2b)}{L_{BF}}\left(-\theta_{BZ}\right) \qquad (60)$$

JOINT G

$$M_{GCX} = 2E\left(\frac{I_X}{L_{CG}}\right)\left[\theta_{CX} + 3\left(\frac{D_{BY}}{L_{CG}}\right)\right] \qquad (61)$$

$$M_{GCY} = 2E\left(\frac{I_Y}{L_{CG}}\right)\left[\theta_{CY} - 3\left(\frac{D_{CX}}{L_{CG}}\right)\right] \qquad (62)$$

$$M^{*}_{GCZ} = \frac{K_1 G (2a)^3 (2b)}{L_{CG}}\left(-\theta_{CZ}\right) \qquad (63)$$

JOINT H

$$M_{HDX} = 2E\left(\frac{I_X}{L_{DH}}\right)\left[\theta_{DX} + 3\left(\frac{D_{AY}}{L_{DH}}\right)\right] \qquad (64)$$

$$M_{HDY} = 2E\left(\frac{I_Y}{L_{DH}}\right)\left[\theta_{DY} - 3\left(\frac{D_{CX}}{L_{DH}}\right)\right] \qquad (65)$$

$$M^{*}_{HDZ} = \frac{K_1 G (2a)^3 (2b)}{L_{DH}}\left(-\theta_{DZ}\right) \qquad (66)$$

TABLE-15 EQUILIBRIUM CONDITIONS AND EQUATIONS
EQUILIBRIUM OF MOMENTS AT JOINTS

$\Sigma M_{AX} = M_{ABX} + M_{ARX} + M^{*}_{A[illegible]} = 0$	①	$\Sigma M_{DX} = M_{DBX} + M_{DHX} + M^{*}_{DGX} = 0$	⑩
$\Sigma M_{AY} = M^{*}_{ABY} + M_{ARY} + M_{A[illegible]} = 0$	②	$\Sigma M_{DY} = M^{*}_{DAY} + M_{DHY} + M_{DGY} = 0$	⑪
$\Sigma M_{AZ} = M_{AQZ} + M^{*}_{A[illegible]} + M_{ABZ} = 0$	③	$\Sigma M_{DZ} = M_{DAZ} + M^{*}_{D[illegible]} + M_{DGZ} = 0$	⑫
$\Sigma M_{BX} = M_{BCX} + M_{BFX} + M^{*}_{BAX} = 0$	④	$\Sigma M_{QX} = M_{QAX} + M_{QUX} + M^{*}_{Q[illegible]} = 0$	⑬
$\Sigma M_{BY} = M^{*}_{BCY} + M_{BFY} + M_{BAY} = 0$	⑤	$\Sigma M_{QY} = M^{*}_{Q[illegible]} + M^{*}_{QDY} + M_{Q[illegible]} = 0$	⑭
$\Sigma M_{BZ} = M_{BFZ} + M^{*}_{BCZ} + M_{BAZ} = 0$	⑥	$\Sigma M_{QZ} = M_{Q[illegible]} + M_{QDZ} + M_{Q[illegible]} = 0$	⑮
$\Sigma M_{CX} = M_{CBX} + M_{C[illegible]} + M^{*}_{CDX} = 0$	⑦	$\Sigma M_{RX} = M_{REX} + M_{RQX} + M^{*}_{RAX} = 0$	⑯
$\Sigma M_{CY} = M^{*}_{CBY} + M_{CGY} + M_{CDY} = 0$	⑧	$\Sigma M_{RY} = M^{*}_{REY} + M^{*}_{R[illegible]} + M_{R[illegible]} = 0$	⑰
$\Sigma M_{CZ} = M_{CBZ} + M^{*}_{CGZ} + M_{CDZ} = 0$	⑨	$\Sigma M_{RZ} = M_{REZ} + M_{RQZ} + M_{RQZ} = 0$	⑱

A

TABLE-15 CONT'D. EQUILIBRIUM CONDITIONS AND EQUATIONS

EQUILIBRIUM OF SHEARS AT JOINTS

$$\Sigma S_{AY} = + \frac{M_{AEX} + M_{EAX}}{L_{AE}} + \frac{M_{DHX} + M_{HDX}}{L_{DH}} - \frac{M_{ABZ} + M_{BAZ}}{L_{AB}} - \frac{M_{QDZ} + M_{DQZ}}{L_{DQ}} - \frac{M_{DCZ} + M_{CDZ}}{L_{CD}} = 0 \quad (19)$$

$$\Sigma S_{BY} = + \frac{M_{BFX} + M_{FBX}}{L_{BF}} + \frac{M_{CGX} + M_{GCX}}{L_{CG}} + \frac{M_{BAX} + M_{ABX}}{L_{AB}} + \frac{M_{BRZ} + M_{RBZ}}{L_{BR}} + \frac{M_{DCZ} + M_{CDZ}}{L_{CD}} = 0 \quad (20)$$

$$\Sigma S_{CX} = - \frac{M_{CGY} + M_{GCY}}{L_{CG}} - \frac{M_{DHY} + M_{HDY}}{L_{DH}} + \frac{M_{CRZ} + M_{RCZ}}{L_{CR}} + \frac{M_{DQZ} + M_{QDZ}}{L_{DQ}} = 0 \quad (21)$$

$$\Sigma S_{RX} = + \frac{M_{QAZ} + M_{AQZ}}{L_{AQ}} - \frac{M_{QDZ} + M_{DQZ}}{L_{DQ}} + \frac{M_{RBZ} + M_{BRZ}}{L_{BR}} - \frac{M_{RCZ} + M_{CRZ}}{L_{CR}} = 0 \quad (22)$$

$$\Sigma S_{BX} = - \frac{M_{BFY} + M_{FBY}}{L_{BF}} - \frac{M_{ABY} + M_{BAY}}{L_{AB}} - \frac{M_{BRZ} + M_{RBZ}}{L_{BR}} - \frac{M_{AQZ} + M_{QAZ}}{L_{AQ}} = 0 \quad (23)$$

$$\Sigma S_{QZ} = - \frac{M_{QAX} + M_{AQX}}{L_{AQ}} + \frac{M_{QDX} + M_{DQX}}{L_{DQ}} + \frac{M_{QRY} + M_{RQY}}{L_{QR}} = 0 \quad (24)$$

$$\Sigma S_{RZ} = - \frac{M_{RBX} + M_{BRX}}{L_{BR}} + \frac{M_{RCX} + M_{CRX}}{L_{CR}} - \frac{M_{RQY} + M_{QRY}}{L_{QR}} \quad (25)$$

B

3. Check solution by substitution

As mentioned before no attempt has been made to solve the simultaneous equations set up in Table 15. Instead, the author substituted the computed values obtained from the three-dimensional moment-distribution solution to find the slopes and deflections. These slopes and deflections are given in the following table.

Joint	A	B	C	D	Q	R
θ_X	+41.049	+16.701	-13.314	-31.977	+12.437	+ 4.8433
θ_Y	+10.299	- 9.5300	- 6.7392	+ 7.5428	+49.280	-42.900
θ_Z	- 0.34398	- 0.34398	- 0.34398	- 0.34398	- 0.27102	- 0.27102

$D_{BX} = 0$; $D_{CX} = +56.817$; $D_{RX} = +24.298$; $D_{AY} = -295.70$; $D_{BY} = -152.62$; $D_{QZ} = +3572.8$; $D_{RZ} = +1347.2$.

Then these values of the slopes and deflections were substituted into the moment and torque expressions to compute all the moments and torques. The following three tables contain a comparison of the moments and torques at the ends of all members in Frame B as determined by the moment-distribution method and by the slope-deflection method.

The author believes that the close agreement of these final results is conclusive evidence of the correctness of the two procedures he has developed in this thesis.

It should be noticed that the values of rotation θ in radians and of deflections D in inches have been multiplied by E, the modulus of elasticity.

Comparison of Moments and Torques at Joints A, B, C and D Computed by Moment-Distribution (M-D-M) and Slope-Deflection (S-D-M)

Joint A			Joint B			Joint C			Joint D		
	MDM	SDM		MDM	SDM		MDM	SDM		MDM	SDM
M_{AQX}	-1002.8	-1002.8	M_{BRX}	-225.9	-225.9	M_{CRX}	+249.9	+245.4	M_{DQX}	+906.7	+895.2
M_{AQY}	- 497.4	- 497.4	M_{BRY}	+425.8	+425.8	M_{CRY}	+308.5	+307.7	M_{DQY}	-356.4	-355.2
M_{AQZ}	- 5.2	- 5.2	M_{BRZ}	- 5.2	- 5.2	M_{CRZ}	- 7.0	- 4.9	M_{DQZ}	- 7.0	- 4.9
M_{AEX}	+ 878.9	+ 878.9	M_{BFX}	+349.8	+349.8	M_{CGX}	-345.1	-345.0	M_{DHX}	-811.5	-811.5
M_{AEY}	+ 238.4	+ 238.4	M_{BFY}	-220.6	-220.6	M_{CGY}	-169.7	-169.7	M_{DHY}	+161.0	+160.9
M_{AEZ}	- 1.4	- 1.4	M_{BFZ}	- 1.4	- 1.4	M_{CGZ}	- 1.4	- 1.4	M_{DHZ}	- 1.4	- 1.4
M_{ABX}	+ 123.9	+ 124.2	M_{BAX}	-123.9	-124.2	M_{CDX}	+ 95.2	+ 95.2	M_{DCX}	- 95.2	- 95.2
M_{ABY}	+ 259.0	+ 259.4	M_{BAY}	-205.2	-205.3	M_{CDY}	-138.8	-139.1	M_{DCY}	+195.4	+196.6
M_{ABZ}	+ 6.6	+ 7.9	M_{BAZ}	+ 6.6	+ 7.9	M_{CDZ}	+ 8.4	+ 7.9	M_{DCZ}	+ 8.4	+ 7.9

Comparison of Moments and Torques at Joints E, F, G and H Computed by
Moment-Distribution (M-D-M) and Slope-Deflection (S-D-M)

Joint E			Joint F			Joint G			Joint H		
	MDM	SDM		MDM	SDM		MDM	SDM		MDM	SDM
M_{EAX}	+403.8	+403.8	M_{FBX}	+156.5	+156.5	M_{GCX}	-191.0	-190.1	M_{HDX}	-441.4	-441.4
M_{EAY}	+119.2	+119.2	M_{FBY}	-110.3	-110.3	M_{GCY}	- 91.7	- 91.7	M_{HDY}	+ 73.7	+ 73.6
M_{EAZ}	+ 1.4	+ 1.4	M_{FBZ}	+ 1.4	+ 1.4	M_{GCZ}	+ 1.4	+ 1.4	M_{HDZ}	+ 1.4	+ 1.4

Comparison of Moments and Torques at Joints Q and R
Computed by Moment-Distribution (M-D-M) and
Slope-Deflection (S-D-M)

	Joint Q			Joint R	
	MDM	SDM		MDM	SDM
M_{QAX}	-2679.3	-2679.3	M_{RBX}	-920.7	-920.6
M_{QAY}	+ 497.4	+ 497.4	M_{RBY}	-425.8	-425.8
M_{QAZ}	- 3.3	- 3.3	M_{RBZ}	- 3.3	- 3.3
M_{QDX}	+2640.7	+2630.1	M_{RCX}	+959.3	+954.7
M_{QDY}	+ 356.4	+ 355.2	M_{RCY}	-308.5	-307.7
M_{QDZ}	- 6.3	- 3.6	M_{RCZ}	- 6.3	- 3.6
M_{QRX}	+ 38.6	+ 38.7	M_{RQX}	- 38.6	- 38.7
M_{QRY}	- 853.8	- 855.9	M_{RQY}	+734.3	+733.6
M_{QRZ}	+ 9.6	+ 10.2	M_{RQZ}	+ 9.6	+ 10.2

IV. LITERATURE CITED

1. Cross, H. Analysis of Continuous Frames by Distributing Fixed-End Moments. Proceedings, American Society of Civil Engineers. 56: 919-928. 1930.

2. Maney, G. A. Secondary Stresses and Other Problems in Rigid Frames: A New Method of Solution. University of Minnesota. Studies in Engineering, 1:1-17. 1915.

3. Cross, H. and Morgan, N. D. Continuous Frames of Reinforced Concrete. p. 226. John Wiley & Sons, Inc., New York. 1932.

4. Ibid. p. 217.

5. American Society of Civil Engineers. Wind Bracing in Steel Buildings, Sixth Progress Report of Sub-Committee No. 31, Committee on Steel of the Structural Division. Proceedings, American Society of Civil Engineers. 65:993. 1939.

6. Timoshenko, S. Theory of Elasticity. p. 233. McGraw-Hill Book Company, Inc., New York. 1934.

7. Ibid. p. 249.

8. Bethlehem Steel Company, Bethlehem Manual of Steel Construction. pp. 279-289. Bethlehem, Pa. 1934.

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VI. VITA

I was born in a small village surrounded by farm lands about twenty miles from Nanchang, the provincial capital city of Kiangsi, China, in the year 1906. The date of birth cannot be given exactly because it was recorded according to the old Chinese calendar.

My father's name was Teh-Tsai Lo, who was the one to have saved my family from poverty. It was because of his savings that I could go on to college after his death.

My early education has been very poor, since I have spent almost all of my childhood on the farm. In fact at the age of fourteen, the time I left the farm, I did not even know the simplest arithmetic.

It was very difficult for me when I was first brought to the modern school. With the anxiety to learn and the anxiety to catch up the lost time, I studied day and night.

During my high school years I was compelled to leave school twice on account of illness and once on account of war. It amounts to almost three years. Somehow I managed to put myself together to study by myself. I have studied algebra, geometry, trigonometry, etc., without a teacher.

Ever since I was old enough to appreciate the modern education I always admired the scientists and engineers. My inclination toward the engineering profession had already been deeply rooted in my early teens. I received the degree of Bachelor of Science in Civil Engineering from Fuh Tan University, Shanghai, China.

After my graduation I taught school for four years. By taking the advantage of the government support I came to the United States. I received the degree of Master of Science from the University of Michigan in the field of transportation engineering.

By the influence of Timoshenko's lecture, I began to be interested in structural theories. In the fall of 1938 I came to Iowa State College.

With the good foundation Professor Frank Kerekes has given me, I began to develop a new field of research, the continuous space frames. I was elected to membership in Phi Kappa Phi in recognition of my work in the year 1941.